WHAT IS LOGIC?

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1 LOGIC AND INFERENCE

It is far from clear what is meant by logic or what should be meant by it. It is nevertheless reasonable to identify logic as the study of inferences and inferential relations. The obvious practical use of logic is in any case to help us to reason well, to draw good inferences. And the typical form the theory of any part of logic seems to be a set of rules of inference.

This answer already introduces some structure into a discussion of the nature of logic, for in an inference we can distinguish the input called a premise or premises from the output known as the conclusion. The transition from a premise or a number of premises to the conclusion is governed by a rule of inference. If the inference is in accordance with the appropriate rule, it is called valid. Rules of inference are often thought of as the alpha and omega of logic. Conceiving of logic as the study of inference is nevertheless only the first approximation to the title question, in that it prompts more questions than it answers. It is not clear what counts as an inference or what a theory of such inferences might look like. What are the rules of inference based on? Where do we find them? The ultimate end one tries to reach through a series of inferences is usually supposed to be a true proposition. Frege [1970, 126] wrote that "the word 'true' characterizes logic." But how does this desideratum determine the rules of inference? A few distinctions will illustrate the embarrassment of riches covered by the term "logic" and at the same time sharpen the issues.

Inferences can be either deductive, that is, necessarily truth preserving, or ampliative, that is, not necessarily truth preserving. This distinction can be identified with the distinction between such steps in reasoning as do not introduce new information into one's reasoning and such as do not do so. For if that information is genuinely new, its truth cannot be guaranteed by the old information. Or, rather, we could thus identify deductive inferences as uninformative ones if we had a viable notion of information at our disposal. Unfortunately, the viability of a notion of information that could be used to make the distinction has been challenged by philosophers, notably by W.V. Quine [1970, 3-6, 98-99], as a part of his criticism of what he calls the analytic-synthetic distinction [Quine, 1951]. A closer examination shows that we have to distinguish from each other two kinds of information, called depth information and surface information [Hintikka, 1970; forthcoming (c)]. A valid deductive inference cannot introduce new depth information, but it can

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increase surface information. Quine's scruples can be best understood as being based on the difficulty of separating depth information and surface information purely behaviorally.

Philosophers generally consider deductive reasoning as the paradigmatic type of inference. However, there is a widespread view among nonphilosophers that might be called the Sherlock Holmes view. According to it all good reasoning including ampliative reasoning, turns on "deductions" and "logic". (Cf. [Hintikka, 2001].)

2 DEDUCTIVE INFERENCE AND INFORMATION

Being truth-preserving, deductive inference is essentially cumulative or, to use a more common term, monotonic. In recent years, spurred largely by studies of automated reasoning of different kinds, there has mushroomed a wide variety of nonmonotonic logics. (See e.g. [Antoniou, 1997; Gabbay *et l.*, 1993-1996, vol.3].) They are all obviously modes of ampliative reasoning, and should be studied as such. They will be commented on in §17 below.

If a deductive inference does not introduce new information, it is in some sense uninformative or tautological. Such uninformativeness of deductive inference was maintained by among others by the early Wittgenstein and by the logical positivists. (See [Hempel, 1945; Dreben and Floyd, 1991].) Such a view is unsatisfactory, for it leaves unexplained what the reasoner gains by drawing such allegedly uninformative conclusions. Something important can obviously be gained through deduction. This air of unintuitiveness can be dispelled by means of the distinction between depth information and surface information mentioned in §1. Surface information can be characterized as the information that can be read off from a sentence without any nontrivial deductive ado, while depth information is the totality of (surface) information that can be extracted from it by deductive logic. The emergence of new surface information explains why purely deductive inference can be experienced as informative and even productive of surprises.

The noninformativeness (in the sense of depth information) of deductive inference is a presupposition of one of the most important kinds of application of logic. This application is axiomatization. The leading idea of this method, vividly emphasized by David Hilbert, is to summarize our knowledge of some model or class of models, for instance certain physical systems, in an axiomatic theory [Hilbert, 1918]. That model or those models can then be studied by deducing theorems from the axiom system. Such a summation is not possible if new information about the subject matter can be introduced in the course of the deduction.

3 ANALYTIC INFERENCES

A distinction within deductive inferences can be made between those that depend only on the meaning of logical concepts, expressed in language by what is known as logical constants, and those that depend also on the meaning of nonlogical constants. Deductive logic in the narrow sense is restricted to the former. Truth-preserving inferences that are not purely logical in this narrow sense are often called analytic. This distinction presupposes a distinction between logical and nonlogical concepts, expressed by logical and nonlogical terms. It is not obvious how such a distinction can be drawn, except perhaps by merely enumerating the logical constants of the given language. (Cf. here [Etchemendy, 1983; 1990; Hintikka, 2004b].) (Concerning the different meanings of "analytic" in logic and its history, see [Hintikka, 1965; 1973].)

Deductive inferences involving nonlogical words are studied among other places in what is known tendentiously as philosophical logic. (This label is misleading because it suggests that deductive logic in the narrower sense is not philosophically important.) Historically studies in such earliest "philosophical" logic were logics calculated to capture such notions as necessity and possibility. They are known as modal logics. (See e.g. [von Wright, 1951a; Carnap, 1947; Kripke, 1963; Lemmon and Scott, 1977; Shapiro, 1998; Blackburn *et al.*, 2001].) Later, attempts have been made to capture such related notions as entailment and relevance. (Cf. e.g. [Anderson *et al.*, 1992].) Philosophical logic also comprises among its other branches epistemic logic, that is, the theory of inferences that depend on the meaning of epistemic terms (see [Hintikka, 1962; 2003]), deontic logic (theory of normative discourse (see [von Wright, 1951b; 1983; Hilpinen, 1971])), and tense logic (theory of inferences turning on the meaning of temporal terms (see [van Benthem, 1982; Gabbay *et al.* 1993–1996, vol. 4])).

The logical behavior of many terms that do not overtly deal with truth-preserving inferences can still be studied by the same method and often partly reduced to the theory of truth-preserving logics. Cases in point are the logic of questions and answers (erotetic logic) which can be considered an aspect of epistemic logic. This is made possible by the fact that questions can be construed as requests for information and their behavior can be studied by reference of the statement (known as the desideratum of a question) which specifies the epistemic state that the questioner wants to be brought about. Similar things can be said of the logic of commands.

Borderline cases are logics that depend on the meaning of probabilistic terms (probability logic [Adams, 1998]), or the behavior of the membership relation (set theory), or on the part-whole relation (mereology (see [Husserl, 1913; Fine, 1995; Smith, 1982])). Indeed, attempts have been made, among others by Rudolf Carnap [1950; 1952], to interpret probability (in at least one of its senses) as a purely logical concept. An especially difficult question concerns set theory. Some of the most characteristic axioms of set theory, for instance the axiom of choice, can be construed as logical truths of higher-order logic (see below). Yet in set theory models are sometimes studied in which the axiom of choice is not true. In the usual foundations of set theory the axiom of choice is not treated as a part of the logic used but as an additional mathematical assumption. Thus currently both probability theory and set theory seem to reside in a limbo between logic and special mathematical theories.

4 AMPLIATIVE INFERENCE

Ampliative reasoning includes prominently inductive reasoning, in which the premises are particular propositions and the conclusion either a generalization or a new particular proposition [Skyrms, 1966]. It is to be noted, however, that induction or *epagoge* originally included and sometimes simply meant a different kind of inference, viz. the extrapolation and interpolation of partial generalizations [Hintikka, 1993]. The logic of inductive inference in this historical sense is quite different from that of the ordinary induction which might be labelled "Humean induction". Inductive inference in this sense can be either qualitative or probabilistic. Qualitative induction can depend on the elimination of alternatives to the conclusion (eliminative induction). Probabilistic induction is sometimes thought to subsume inductive inference to probabilistic inference.

Even though induction was earlier thought of as the only alternative to deductive reasoning, an abundance of other kinds of ampliative modes of reasoning have recently been studied. They include the theories of rational belief change and various forms of nonmonotonic reasoning. They will be discussed below in §17.

Besides the usual so-called classical logics there are also logics that do not involve nonlogical concepts but which are supposed to be grounded on a nonstandard interpretation of logical notions themselves. They include intuitionistic and constuctivistic logics. (See [Heyting, 1956; Dummett, 1977] and [Bishop, 1967].) Some logics are supposed to be applicable to special subject matters, as e.g. the so-called quantum logics. (See [Hooker, 1975–77].)

5 LOGICAL VS. EXTRALOGICAL SYSTEMATIZATION

Instead of thinking of logic as the study of inferences, it is sometimes construed as a study of logical truths. The two conceptions are closely interrelated. In order for a deductive inference from F to G to be valid, the conditional sentence If F, then G (in symbols $(F \supset G)$) must be logically true. Whether the converse relation holds depends on whether the validity of $(F \supset G)$ guarantees the existence of a rule of inference that mandates a move from F to G. Indeed, more may be required for the validity of an inference from F to G than the logical truth of $(F \supset G)$. For instance, it may be required that the rule takes us from a way of verifying F to a way of verifying G.

Speaking of logical truths can be misleading, however, for logical truth need not and perhaps should not be considered as a variant of plain ordinary truth. Truth is relative to one model (scenario, possible world), whereas logical truth is on this conception truth in all possible models (scenarios, possible worlds). This distinction is reflected in a parallel distinction between axiomatizations of ordinary substantial theories and axiomatizations of this or that part of logic. What are known as formal systems of logic are systematizations of logical truths. They are formulated in a partial analogy with substantial axiomatic theories. A few formulas or types of formulas are designated as axioms from which theorems are derived according to purely formal rules. This has been the standard format of formal systems of logic since Frege.

There are nevertheless important differences between the two kinds of axiomatizations. What an ordinary axiom system accomplishes (if successful) is to capture all the truths about the relevant system or systems, as deductive consequences of the axioms. In other words, what an ordinary non-logical axiom system is calculated to do is to help us to study some aspect of reality or some mathematical structure (or class of structures) by examining the logical consequences of an axiom system. In contrast, what a formal system of some part of logic is calculated to do is in the first place merely to list mechanically all the logical truths of that part of logic. This is a substantial difference. It is for instance not always the case that logical consequence relations can be captured by purely formal (mechanical) rules of deductive inference. Furthermore, the so-called rules of inference of a logical axiomatization need not ever be truth-preserving, unlike the real rules of inference of a substantial axiom system. They are not necessarily intended as guidelines for actually drawing inferences. In a sense, only the rules of inference of a nonlogical (substantial) axiom system are genuine rules of logical inference. Hence the role of logic as a system of rules of inference is in a sense more prominent in nonlogical axiomatizations than in logical ones.

Likewise, the exhaustiveness of the enumeration, known as completeness of the axiom system, is different in the two kinds of axiomatization. In the case of logical axiomatization, the term "semantic completeness" is typically used, whereas in the case of substantial axiom systems one usual term is "deductive completeness."

6 SYNTAX VS. SEMANTICS

The concept of logical truth becomes useful when we begin to develop an explicit logical theory, maybe even an axiomatic one. Such a systematic logical theory can be implemented in two main ways. One leading idea in the early development of modern logic was to try to invent a notation or symbolism so explicit and so flexible that the criteria of valid inference and of logical truth could be captured by reference to the formal features of this symbolism, without referring to their meaning. In other words, the application of these criteria happens by a calculation with symbols. This idea is what the term "symbolic logic" was calculated to highlight. Such a study of the purely formal features of a language is usually referred to as its syntax. In systematic logical studies this approach includes what is known as proof theory, and is sometimes referred to as such. This idea is somewhat similar to the project of transformational grammarians like Noam Chomsky to capture what are essentially semantical notions, such as ambiguity, co-reference and logical form, in purely syntactical terms [Chomsky, 1975].

In contrast, a study of the relations of a language to the reality it can be used to represent is known as its semantics. In logic, it includes what is known as model theory, even though this term usually refers to the particular kind of studies launched largely by Alfred Tarski [1935; 1944, 1956]. Sometimes these two are distinguished from pragmatics, which is supposed to be the study of the uses of language. This distinction is not well defined, however, for the language-world relations studied in semantics can be constituted by rule-governed uses of language, as for instance in Wittgenstein's language games or in game-theoretical semantics [Wittgenstein, 1953; Hintikka and Sandu, 1997].

One of the most important semantical concepts is that of truth. Since the validity of a deductive rule of inference means that is is necessarily truth preserving, the theory of deductive logic must have its foundation in a theory of semantics (model theory). At the earlier stages of the development of modern logic, the need of such a basis was usually denied or at least neglected. The hope was that in an appropriate symbolism all the important semantical properties and relations between propositions would have syntactical counterparts that could be studied proof-theoretically, for instance by means of suitable formal systems of logic.

7 THE LIMITS OF SYNTACTICAL APPROACHES

This hope was in the case of logical notions shattered by the incompleteness theorems of Kurt Gödel and the impossibility theorems of Alonzo Church and Alfred Tarski. (See [Davis, 1965].) Gödel showed that any first-order system of elementary arithmetic is inevitably incomplete in that some arithmetical propositions in it are true but unprovable. He also showed that the consistency of a system of elementary arithmetic cannot be proved in the same system. Church showed that the decision problem for first-order logic is undecidable, that is, that the set of logical truths of this part of logic is not recursive. Tarski developed techniques of defining the concept of truth for suitable logical languages, but at the same time showed that such definitions can only be formulated in a richer metalanguage [Tarski, 1935]. Since our actual working language presumably is the richest one we have at our disposal, the concept of truth cannot according to Tarski be consistently applied in what Tarski called the "colloquial language". In particular, Tarski showed that the concept of truth for any first-order axiom system cannot be defined in the same system.

These results are also relevant to the question of the relationship of syntactically explicit ("formalized") logic and the logic implicit in natural languages and in ordinary discourse. Earlier it was often thought and said that the logical symbolism is little more than a streamlined and regimented version of an ordinary language. The main advantage of a symbol language was seen in its freedom of ambiguities and other obscurities.

This idea lives on in a modified form in that some theorists of language consider the notion of logical form, expressible in the logical symbolism, as playing an important role in grammatical theory.

There is strong evidence, however, to suggest that logical symbolism and ordinary language are more radically different than was assumed by the earliest "symbolic logicians", For instance, the presuppositions of first-order language examined in SS11–12 below are fairly obviously not satisfied by our ordinary language. Likewise, the notion of independence discussed in §12 is not expressed in most natural languages in a uniform way, even though it turns out to be quite prevalent in their semantics.

The wisest course appears to be to consider formal logical languages as alternatives to one's own natural language in the same sense in which one's second language offers an alternative way of expressing oneself. What can be hoped is that there might be a privileged kind of logical language that would express the true logical forms underlying all languages, formal and natural. This ideal language would be a true "language of thought" alias Frege's *Begriffsschrift* or conceptual notation [1879]. Whether our actual logical languages can claim to be approximations to such a language of thought, and if so to what extent is a moot question.

8 THE LIMITS OF SEMANTICAL APPROACHES

The need of a semantical basis of a theory of deductive logic has nevertheless been denied for different reasons. One reason is the view which was strongly represented among the early major figures of contemporary logic (Frege, early Russell, early Wittgenstein etc.) and which was still current recently (Quine, Church etc.) to the effect that the semantics of a language cannot be expressed in the same language. This kind of view has been strongly encouraged by the result of Alfred Tarski [1935] according to which the central semantical notion of truth can be defined for a first-order language (see below) only in a richer metalanguage. In so far as this result applies to our actual working language, it suggests that the notion of truth cannot play a role in our "colloquial language" as Tarski called it, for it cannot have any richer metalanguage above it.

If this impossibility really prevails, there cannot be any general semantical foundation of logic understood as a theory of inference.

How conclusive are these negative results in a general perspective? This question will be revisited below. In any case, some parts of logic allow for a complete axiomatization. For one thing, Gödel proved the semantical completeness of the received first-order logic in 1930. Since this logic is often taken to be the central area of logic — and even taken to exhaust the purview of symbolic logic —, this has created the impression that the dream of purely symbolic deductive logic is indeed realizable and that the negative results pertain only to impure extensions of the proper province of deductive logic. In any case, the questions whether different parts of logic admit of a semantically complete axiomatization and whether different mathematical theories admit of a deductively complete axiomatization are crucial in establishing the prospects and the limits of symbolic (syntactical) methods in logic. This is why Gödel's first incompleteness theorem is so important, in that it shows that elementary arithmetic cannot be completely axiomatized in the sense of deductive completeness.

The question of axiomatization should not be confused with the question whether logic is formal discipline. By one commonly used definition, in deductive logic a sentence is logically true if and only if its truth depends only on the way in which logical terms occur in it. If so, logical truths of deductive logic are all formal independently of the question of whether they can be mechanically enumerated by means of a formal axiom system.

9 STRATEGIC VS. DEFINITORY RULES

An important further distinction between different aspects of logical studies derives from the nature of logical reasoning (inference-drawing) as a goal-directed activity. In practically all such activities a distinction can be made between different two kinds of rules. This distinction is especially clear in the case of games of strategy. (Cf. [von Neumann and Morgenstern, 1944].) In them, we can distinguish the definitory rules which specify what may happen in the game from the strategic rules which tell how to play the game better or worse. For instance, the definitory rules of chess determine what moves are possible, what counts as checking and checkmating etc., whereas following the strategic rules of chess is what makes a player better or worse. Strategic rules are not merely heuristic. They can in principle be as precise as the definitory rules, even though they are quite often so complicated as to be impossible to formulate explicitly.

Logical theory involves the study of both definitory rules and strategic rules of the "game" of logic. (See [Hintikka, 2001].) It is important to realize that what are called rules of inference must be considered as definitory rules. They do not tell what inferences one should draw from the available premises or what inferences people in fact draw from them always or usually. They are neither descriptive nor prescriptive rules; they are permissive ones. They tell what one can do without committing a logical mistake. They tell one how to avoid logical mistakes, known as fallacies. The study of fallacies has been part of logic ever since Aristotle. In so far as fallacies are violations of rules of inference in the narrow sense, the study of fallacies is not an independent part of logical studies. Many of the traditional fallacies are in fact not mistakes in applying deductive rules of inference, but either violations of the rules of other parts of logic or else strategic mistakes. For instance, the fallacy of begging the question did not originally mean circular reasoning, as it is nowadays viewed, but the mistake of asking directly the principal question of an interrogative inquiry. This question is supposed to be answered by putting a number of "small" questions to suitable sources of information.

This definitory vs. strategic distinction provides an interesting perspective on logical studies. In logical theorizing, a lion's share of attention has been devoted to definitory rules at the expense of strategic rules, even though the latter ones are what defines good reasoning. One historical reason for this may be the crisis in the foundations of mathematics in the early twentieth century which prompted an emphasis on the soundness of rules of logical inference. Yet for actual applications of logic strategic rules are incomparably more important.

There also prevails a confusion as to how certain types of human reasoning should be modeled by means of a logical system. Are the principles of such human reasoning to be captured by means of definitory rules or of strategic rules of reasoning? The latter answer is the more promising one, even though many developments in logic are apparently predicated on the former answer. For instance, it is fairly obvious that the principles of rational belief change should be captured by strategic rules rather than definitory rules, contrary to what many theorists are trying to do.

A characteristic difference between the two kinds of rules is that definitory rules normally concern particular moves, whereas strategic evaluation pertains in the last analysis to entire strategies in the sense of game theories. (Such strategies prescribe what a player should do in all the possible situations that he or she or it might encounter in a play of the game.)

These observations help us to understand what can correctly be meant by logical necessity. Contrary to what the dominating view of most philosophers was for a long time, there is no necessity about actually drawing logical inferences. Even when G is a logical consequence of F, one does not have to think of G as soon as one thinks of F. What is the case is that G must be the case as soon as F is true. You cannot bring it about that F without ipso facto bringing it about that G. Hence it is permissible in truth-preserving reasoning to move from F to G.

The definitory vs. strategic distinction also throws some light on the idea of nonmonotonic reasoning. The aim of logical reasoning manifesting itself in the form of a series of inferences, is normally a true ultimate conclusion (if for a moment we look away from probabilistic inferences). Hence it might seem as if the very idea of logical inferences that do not always preserve truth were an oxymoron. A solution to this problem is to point out that a sequence of inferences may ultimately lead to a true ultimate conclusion even if some of the individual inferences in the sequence are not truth preserving. This observation can be taken to vindicate the possibility of non-monotonic logics. At the same time it shows that a satisfactory theory of nonmonotonic logic must contain a strategic component, for rules of individual inferences cannot alone guarantee the truth or the probability of the conclusion of an entire sequence of nonmonotonic inferences. Such a strategic component is nevertheless missing from most nonmonotonic logics.

10 WHAT IS FIRST-ORDER LOGIC?

One way of trying to answer the title question of this article is to examine what has generally been taken to be the core area of logic. This is the logic of propositional connectives and quantifiers, known as first-order logic, predicate logic or quantification theory. (Cf. [Smullyan, 1968].) Since these are logical notions par excellence, in the light of the distinctions just made, this core area should be the study of deductive logic based on the meaning of logical concepts, studied primarily from the vantage point of correctness (definitory rules). Now the central logical concepts are generally recognized to be the (standard) quantifiers, that is, the notions expressed by "all" and "some" and propositional connectives like "and", "or", "if-then", and "if and only if". Some of the features of the behavior of quantifiers were studied already in the Aristotelian syllogistic which dominated logic until the nineteenth century. Gottlob Frege and C.S. Peirce, by removing the restriction to one-place predicates as distinguished from relations, overcame many of the limitations of syllogistic logic. The result, codified in the *Principia Mathematica* (1910-13) by Bertrand Russell and A.N. Whitehead, is a general theory of quantifiers and propositional connectives. Slowly, this theory was divided into first-order logic, in which the values of quantified variables are individuals (particulars) and higher-order logic, in which they can be entities of a higher logical type, such as sets, properties and relations. It is this first-order logic that is generally considered as the core area of logic.

What first-order logic is like is most directly explained by identifying the structures that can be discussed by its means. In an application of first-order logic, we are given a class of individuals, known variously as the domain of the model or its universe of discourse. In the inference rules for first-order logic, they are thought of as particular objects. They are represented by individual constants (names of individuals). On that domain, a number of properties and relations are defined, represented by predicates of one or more argument places. The central notions are the two quantifiers, viz. the existential quantifier $(\exists x)$ and the universal quantifier $(\forall y)$, each of which has a variable attached to it facilitating a later reference to the same individual. They express the notions of *at least one* and *every* as applied to the domain of individuals. Furthermore, the logical notation includes a number of propositional connectives, for instance negation $\sim$, conjunction &, disjunction $\vee$, and conditional ("if – then") $\supset$. In classical logic, they are assumed to be characterized by their truth-tables. Atomic formulas are formed by inserting individual constants and/or individual variables into the argument-places of predicates. The rest of the formulas are formed by repeated uses of quantifiers and propositional connectives. Often, the notion of identity is also included, expressed by $=$ with individual constants and/or variables flanking it. Formulas without free variables are called sentences. They express propositions about the universe of discourse.

First-order logic has a regular model theory. Among its metatheoretical properties there are compactness, the Löwenheim-Skolem properties, and separation properties. Most importantly, first-order logic admits of a semantically complete axiomatization, as was first shown by Gödel in 1930. These features of first-order logic are sometimes taken to be characteristic of logical systems in general. First-order logic was often considered by earlier philosophers as *the* logic, and even later their idea of what logic is, is modeled to a considerable extent on first-order logic.

11 PRESUPPOSITIONS OF THE RECEIVED FIRST-ORDER LOGIC

The central role of first-order logic is illustrated by the fact that it is widely used as a medium of representing the semantical (logical) form of natural-language sentences, including philosophical theses. Even Chomsky at one time [1986] considered his LF structures, which are the basis of semantical interpretation of English sentences, have a structure closely similar to the forms of a first-order (quantificational) formula. This practice is not without problems, however. One of the most prominent features of the notation of received first-order logic is a difference between it and the way natural language apparently operates. Natural-language verbs for being do not have a single translation into the logical notation, but have to be expressed in it differently in their different uses. (Cf. here [Hintikka, 1979].) A verb like *is* must on different occasions be treated as expressing identity, predication, or existence. On still other occasions, it may help to express class-inclusion, location or assertion (veridical *is*). Does this show that verbs like *is* are ambiguous? Only if the received notation of first-order logic is assumed to be the uniquely correct framework of semantical representation. Even though some linguists are in effect making this assumption, it is not at all obvious. It is not difficult to devise a semantic representation for English quantifiers that does not incorporate the ambiguity assumption and to formalize such a representation. In philosophy, no thinker before the nineteenth century assumed the triple ambiguity of verbs for being, that is, the alleged identity-predication-existence ambiguity.

One of the critical ingredients of first-order logic is the notion of the domain of individuals. This is a genuine novelty of modern logic. Earlier logicians admittedly used quantifiers, but for them these quantifiers were expressed by what Russell called denoting phrases (like "some Greek" or "every European") which meant that each quantifier ranged only over a definite restricted class of values. In order to understand such a denoting phrase, aka quantifier phrase, one did not have to know what the entire universe of discourse was. This in effect made the notion of a universe of discourse redundant. Moreover, for Aristotle, the idea of a mind-independent universe of discourse would have not made any sense also because he considered realizations of forms in the soul on a par with their realizations in external reality.

Some philosophers have thought that there is only proper application of logic, viz. the actual world. For them, the determination of the basic objects of reality becomes a major philosophical problem. A more realistic view takes the applications of logic to concern sufficiently isolated parts of some actual or possible world, in analogy with probability theorists' sample space points or physicists' "systems". (See [Hintikka, 2003a].) Some philosophical logicians use chunks of the actual world called "situations" as individuating applications of logic. (See [Barwise, 1981; 1989].)

The truly important novelty of first-order logic (or more generally, logic of quantifiers) does not lie in the ambiguity thesis, but in its power to express functional dependencies (dependencies between variables). Indeed, the entire first-order logic is equivalent with a quantifier-free system where all variables are universal and in which the job of existential quantifiers is done by function constants.

12 THE MEANING OF QUANTIFIERS

But is first-order logic in its received form fully representative of what logic is? The answer must turn on the meaning of the logical constants of first-order logic, especially of quantifiers and of identity. The common assumption is that quantifiers receive their meaning from their "ranging over" a class of values. An existential quantifier $(\exists x)$ prefixed to an open formula $F[x]$ says that the class of such values x satisfying $F[x]$ is not empty, and $(\forall x)F[x]$ says that all such values x satisfy $F[x]$. This assumption is among others made by Frege, who proposed that quantifiers be interpreted a higher-order predicates expressing the nonemptyness or exceptionlessness of a lower-order predicate. This is admittedly an important part of what quantifiers express. However, it is important to realize that quantifiers have another semantical role. By means of the formal dependence of quantifier of (Q_2y) on (Q_1x), we can express that the variable y depends on the variable x, in the sense of concrete factual dependence. This function of quantifiers is highly important, and it can be reduced to the "ranging over" idea only on restrictive further conditions. (See here [Hintikka, 1996; 2002a].)

But how is the formal dependence of (Q_2y) on (Q_1x) expressed? In the received logic, by the fact that (Q_2y) occurs in the syntactical scope of (Q_1x). Now in the received first-order logic these scopes are assumed to be nested. Since this nesting relation is transitive and antisymmetric, it cannot express all possible patterns of dependence and independence between variables. Hence the received first-order logic is not fully representative of the function of logical constants. This representative role is best served by the result of removing the restrictions on the patterns of formal dependence between quantifiers. The result is known as independence-friendly (IF) logic. By the same token as the received first-order logic used to be considered the core area of logic, IF logic must now be assigned that role. If so, it is in fact terminologically misleading to use any qualifying word in its name. It is the received first-order logic that deserves a special epithet because of the limitations it is subject to. In spite of this inbuilt bias, the term "IF logic" will be used in the following.

The semantics for this new basic logic is straightforward. The most central semantical concept is that of truth. A first-order sentence is true when there exist suitable "witness individuals" vouchsafing this truth. Thus e.g. $(\exists x)F[x]$ is true if and only if there exists individual b satisfying $F[x]$, and $(\forall x)(\exists y)F[x, y]$ is true if and only if for any individual a there exist b which together satisfy $F[x, y]$. As the latter example shows, such witness individuals may depend on others. The natural truth condition for a quantificational sentence S is therefore the existence of all the functions which produce these witness individuals as their values. (In the extreme case of initial existential quantifiers these functions become constants.) These functions are known as Skolem functions, and the natural definition of truth for S therefore requires the existence of a full array of Skolem functions for S. Such arrays of Skolem functions can be interpreted as winning strategies in certain games of verification and falsification called semantical games, thus throwing some light on how language-world relations are implemented. This "game-theoretical" truth definition is more elementary than Tarski-type ones.

13 SEMANTICAL GAMES

This interpretation of the all-important Skolem functions as codifying winning strategies in certain games opens interesting lines of thought concerning logic in general. The games in question are called semantical games. (See [Hintikka, 1968; Hintikka and Sandu, 1997].) The games in question are called semantical games. Does this interpretation mean that logical theory is part and parcel of the general theory of different kinds of games? The main answer is no. The reason is that the range of games relevant to the semantics of logic is fairly restricted. As an example we may consider how the information sets are determined that are presupposed in semantical games. The information set of a player making the move connected with a certain node of the game tree is the set of nodes in which the player in question can be at that time to the best of that player's knowledge. In semantical games the only important information sets are those due to the relevant player's knowledge and ignorance of what has happened at earlier moves in the same play of the game. Allowing such ignorance is the only thing needed to move from the semantics of the received first-order logic to the semantics of IF first-order logic.

Semantical games admit in fact a simple interpretation. When there is perfect information, such a game (for instance, the game $G(S)$ connected with a sentence S) can be thought of as a kind of game of verification and falsification, albeit not in the most literal sense of the expressions. From the point of the verifier, it is an attempt to find some of the "witness individuals" that would verify S. The falsifier tries to make this task as hard as possible or even impossible. From this idea, the game rules can be gathered without any difficulty. For instance, the first move in $G((\exists x)F[x])$ is the verifier's choice of an individual a from the domain. The game is then continued as in $G(F[a])$. The first move in $G((S_1 \& S_2))$ is the falsifier's choice of S_1 or S_2, say S_o. The game is then continued as in $G(S_o)$.

Speaking of the verifier's choice of an individual is somewhat misleading here, for in an actual play of a game a successful move would require possibly quite elaborate searching for a right individual. Hence semantical games can also be considered as games of seeking a finding.

In spite of the natural structure of semantical games they are not devoid of subtleties. The verifier wins a play of a semantical game if and only if the play ends with a true atomic sentence. The falsifier wins if it ends up with a false one. However, the truth of a sentence cannot be defined as the verifier's win in the game $G(S)$. Rather, it must be defined as the existence of a winning strategy for the verifier. This distinction can be compared with the distinction between a sentence's being verified and being verifiable.

A philosophical perspective is obtained here by considering semantical games in the same way Wittgenstein considered his language-games, viz., as the basic semantical links between language and reality. If this idea is combined with a transcendental idea reminiscent mutatis mutandis of Kant, our knowledge of logic an be thought of as knowledge the structure and characteristics of semantical games. [Hintikka, 1973, chapter 5.]

Be this as it may, game-theoretical semantics is thus a candidate for the role of a semantical foundation of logic. In such a perspective, logic amounts to something like the study of the structure of the language-games that connect language with reality in the case of logical words.

The role of semantical games as the basic link between language and reality is also illustrated by the fact that game-theoretical semantics can be used to interpret some of the most prominent alternatives to the usual first-order logic. This can happen by restricting the set of strategies that the verifier may use, which means restricting the range of variables for Skolem functions. If they are restricted to constructive functions, we obtain an interpretation of constructivistic logic. If they were restricted to functions that are known in some reasonable sense, the result should be an interpretation of intuitionistic logic.

Game-theoretical semantics thus suggests in any case interesting perspectives relevant to the question, "What is logic?"

14 THE IMPLICATIONS OF INDEPENDENCE-FRIENDLY LOGIC: AXIOMATIZABILITY

These suggestions are reinforced and made more specific by what happens in the showcase of game-theoretical semantics, independence-friendly logic. As was pointed out, Gödel's completeness theorem for the received first-order logic encouraged the idea that our real basic logic is semantically complete. This in turn led some thinkers to consider logic as a general study of formal systems.

If independence-friendly logic really is the authentic basic logic, such ideas must be rejected. For independence-friendly logic is not complete in the sense of there existing a formal proof procedure that in the limit produces all logical truths and only them.

At first sight, the question of axiomatizability might seem to affect only the computational implementation of different logics and more generally also of different mathematical theories. Such appearances notwithstanding, the question of complete axiomatizability makes a difference to our ideas about the nature of logic in relation to mathematics. The currently prevalent view seems to envisage an underlying logic which is semantically complete but too weak to capture all modes of mathematical reasoning. Hence the creative element in mathematics lies not in logic, but in mathematics. Furthermore, stronger principles that are needed in mathematical reasoning are typically thought of as new mathematical axioms, the first place perhaps set-theoretical axioms.

This picture is now being challenged. The alternative view suggested by independence-friendly logic is that a suitable logic can capture all the forms of mathematical reasoning. (What this suitable logic might be is discussed later in this article; see SS16–17.) This is connected with the fact that independence-friendly logic incorporates several important modes of inference that were impossible to express in the received first-order logic and likewise makes it possible to capture mathematically important concepts that would not be defined in the earlier first-order logic. These concepts include equicardinality, infinity and topological continuity. The modes of inference in question include the axiom of choice.

However, IF logic is not completely axiomatizable. Hence creativity is needed for the discovery of increasingly intricate logical truths, and no longer a monopoly of mathematicians as distinguished from logicians. This question affects our ideas about the relation of mathematics and logic and hence our ideas about logic. An important research project known as reverse mathematics is calculated to uncover the set-theoretical assumptions on which different mathematical results are based. (See [Friedman, 1970].) However, the main thrust of this project has been to ask which sets have to exist for a mathematical proof to be able to go through rather than to ask directly what stronger set-theoretical axioms are needed in the proofs. Now the existence of a set implies the possibility of applying the logical principle of tertium non datur to its definiens. Hence the project of reverse mathematics can also be viewed as a study of what applications of logical principles like tertium non datur are needed in actual mathematical proofs.

15 NEGATION AND ITS SIGNIFICANCE

Another feature of independence-friendly logic has likewise important implications for our ideas about logic in general. This feature is the behavior of negation. (See here [Hintikka, forthcoming (b)].) The semantical game rule (or rules) for negation in independence-friendly logic are the same as in the received logic of quantifiers. Yet the resulting negation turns out not to be the contradictory negation that is often thought of as being *the* negation. Instead, we receive a stronger dual negation that does not obey the law of excluded middle.

From a game-theoretical viewpoint this is only to be expected, for the law of excluded middle amounts to assuming that semantical games are determinate. And of course there are plenty of perfectly natural games that are not determinate.

Many philosophers will react to this by saying that what it shows is that independence friendly logic is a "nonclassical" logic. But what is their criterion of a logic's being classical? In view of the "classical" character of the game rules for negation in independence-friendly logic, it appears quite as natural to maintain that this logic shows that the law of excluded middle is not part and parcel of the classical conception of logic.

The absence of tertium non datur means that independence-friendly logic is in some ways closer to intuitionistic logic than the received first-order logic, in spite of its greater expressive power.

Of course we have to introduce also a contradictory negation $\neg$ into the independence-friendly logic over and above the dual negation $\sim$ in order to reach an adequate basic logic. The result will be called the extended independence-friendly first-order logic. The contradictory negation cannot be characterized by game rules however, for the "classical" ones yield the dual one. Hence $\neg$ can occur in extended independence-friendly logic only sentence-initially.

The extended independence-friendly logic has an interesting structure. Algebraically, it has the structure of a Boolean algebra with an operator in Tarski's sense. (See [Jónsson and Tarski, 1951; 1952].) In a sense, this structure is therefore the true algebra of logic. By Tarski's results, the extended independence logic admits to a set-theoretical ("geometrical") interpretation. In this interpretation the strong negation turns out to express a generalization of the notion of orthogonality. In terms of orthogonality, we can even define such notions as dimensions, coordinate representation etc. for purely logical spaces. (See [Hintikka, 2004c; forthcoming (b)].)

In sufficiently strong languages, there must thus be two different notions of negation present, the strong dual negation and a contradictory one. This presumably applied also to natural languages even though in them there usually is but one grammatical construction for negation. This puts the notion of negation in natural languages to a new light.

If we make suitable further assumptions concerning the logical spaces with two negations, we obtain more elaborate algebraic structures which largely remain to be examined. By using game-theoretical ideas, we can also introduce in a natural way probability measures on logical spaces. Whether, and if so in what sense, this makes probability a logical concept remains to be examined.

16 HIGHER-ORDER LOGICS

These developments throw new light also on second-order logic and other higher-order logics. First, however, it is in order to see what these logics are like.

First-order logic is characterized by the fact that all quantifiers range over all the individuals in the given universe of discourse. Second-order logic is obtained when quantifiers are admitted that range over sets and other predicates-in-extension of individuals. They are known as second-order quantifiers and their values second-order entities. Third-order quantifiers range over sets and predicates-in-extensions of second-order entities, and so on. Instead of such extensional entities, the concepts that define them are sometimes employed in higher-order logics as values of higher-order variables. We can thus distinguish extensional and intensional interpretations of higher-order logics. When the different orders are separated from each other, the result is also known as a theory of types in a broad sense.

Within extensionally construed higher-order logics we can distinguish two different interpretations. If the higher-order quantifiers range over all extensionally possible entities of the appropriate lower type, we obtain what is known as the standard interpretations. If they range over some designated subset of such entities, we are dealing with a nonstandard interpretation. The most prominent such nonstandard interpretation is the one in which higher-order quantifiers range over entities definable in the language that is being used. Often, the term "non standard interpretation" is restricted to this one.

The distinction was first formulated and the terms "standard" and "nonstandard" introduced by Leon Henkin in 1950. However, the distinction was clear to Frank

Ramsey who proposed transforming the intensional higher-order system of Russell's and Whitehead's *Principia Mathematica* into a form in which it can be given a standard interpretation. (See [Ramsey, 1925].)

Even earlier, a special case of the standardness idea had played an important role in the foundations of mathematics in the form of the notion of an arbitrary function. The class of all such functions is nothing but the standard range of the function variables of the appropriate type. The important development in the history of mathematics of the idea of an arbitrary function is in effect the development of the standard interpretation of function variables. Applied to functions from natural numbers to integers the notion of arbitrary function becomes the idea of an arbitrary sequence of integers that has played an important role in the general theory of functions, especially in their representations as series of different kinds.

For most mathematical conceptualizations and modes of reasoning, second-order logic with standard interpretation is amply sufficient. Such a logic is not semantically complete, however.

Higher-order logics are in a certain sense parallel to set theory. An axiomatized set theory is like a higher-order logic without type restrictions. The logical principles that are normally used in set theory are those of first-order logic, not of higher-order logic. The question of the adequacy of such first-order axiomatizations of set theory must still be considered as being open. (See [Hintikka, forthcoming (a)].)

The difficulties in this direction are in the first instance due to the fact that the standard interpretation cannot be imposed on the models of a system of set theory by first-order axioms. Hence the idea of a model of an axiomatic first-order set theory can be understood in two different ways. On one interpretation, the would-be membership relation $\in$ is interpreted like any first-order predicate. But we can also require that it be actually interpreted as the membership relation. One source of difficulties here is revealed by the question whether an axiomatic set theory can have models that can be interpreted in the latter way even locally.

The question of the existence of higher-order entities depends on their mode of existence and on their knowability (or lack thereof). Several philosophers have been suspicious of such entities and would rather dispense with them altogether. W.V. Quine among others has for this reason preferred set theory to higher-order logic. This is a questionable position, however, for an attempt to interpret the first-order quantifiers of an axiomatized set theory set-theoretically leads to serious difficulties. (See [Hintikka, 2004a; forthcoming (a)].)

In any case, in either approach assumptions are needed that go beyond first-order logic, either higher-order principles of reasoning or axioms of set existence. A typical example is the axiom of choice which unsurprisingly has been a bone of contention in the foundations of mathematics. Hilbert's epsilon-calculus can be viewed as a large-scale attempt to reduce the axiom of choice to the first-order level. (See [Hilbert and Bernays, 1934–1939].)

17 A DISTINCTION WITHOUT ONTOLOGICAL DIFFERENCE?

Certain recent advances in logic have disturbed the neat distinction between first-order and higher-order logic, however. (See [Hintikka, forthcoming (b)].) For one thing, independence-friendly logic captures on the first order level several important concepts and modes of reasoning that earlier were thought of as being possible to capture only by second-order means. As was noted they include the notions of equicardinality and infinity, the axiom of choice, König's lemma and topological continuity. In the extended independence-friendly logic we can add to the list among others the notions of well-ordering and mathematical induction. Their availability means that we can on the first-order level (that is, without quantifying over higher-order entities) carry out enormously more reasoning, including mathematical reasoning, than was previously thought of as being possible to do on this level. Moreover, this is possible not only without quantifying over higher-order entities but also without evoking the principle of excluded middle, in the sense that this principle is not assumed in independence-friendly logic. However, there are modes of logical reasoning that cannot be so captured and propositions that cannot be expressed in extended independence-friendly logic. The prime examples are offered by prima facie propositions in which the contradictory negation $\neg$ occurs within the scope of a quantifier. Such formulas cannot be assigned an interpretation by means of game-theoretical semantics. They can only be interpreted by reference to the totality of substitution-instances of their non-quantified part, the substitution-values being of course the names of all the members of the domain. When this domain is infinite, this involves an application of tertium non datur to infinite sets as closed totalities. The resulting meanings are as a consequence nonelementary ("infinitistic") in a sense in which independence-friendly logic is not. (Semantical games do not involve infinite sets as closed totalities.)

At the same time the resulting infinitistic logic is first-order in the sense that no quantification over second-order or higher-order entities is involved. It nevertheless turns out that the resulting logic — that is, extended independence-friendly logic reinforced by contradictory negation which is allowed to occur within the scope of quantifiers and moreover allowed to be arbitrarily nested — is as strong as the entire second-order logic. Since second-order logic with standard interpretation is as strong a logic as anybody is likely to need in science or classical mathematics, it is thus seen that all logic we reasonably may need can be done ontologically speaking on the first-order level. Quine has said that to be is to be a value of a bound variable. If that is the case, we don't need the existence of any second-order (or other higher-order) entities. In any case it is true that we need not worry about what the existence of higher-order entities means or which of them must be assumed to exist.

Instead, what distinguishes the kind of reasoning that is now discussed in the context of higher-order logics from elementary reasoning is the unrestricted use of tertium non datur. Such use was in fact considered as the mark of infinitistic reasoning by the likes of David Hilbert and L.E.J. Brouwer [1923]. Whether this means that the contrast between first-order logic and higher-order logic disappears is a mere matter of terminological preference.

18 ALTERNATIVE LOGICS?

All this leaves in the dark the status of the many "alternative logics" that have sprung up in recent years. (See [van Benthem, forthcoming; Ginsberg 1987].) Since they are typically non-monotonic, they cannot be truth-preserving or deductive. But if so, how can they be claimed to be alternative to the traditional systematizations of deductive logic? And if they embody ampliative inference, what is the new information that is brought in and where does it come from? And if they are (as inductive logic was argued to be) dependent on unspoken assumptions, what are those assumptions?

In order to answer such questions, we can consider as an example one of the most clear-cut "alternative logics", developed initially by John McCarthy [1980] and called "inference by circumscription". Its basic idea is clear. The inferences considered in the theory of circumscriptive inference are based not only on their explicit premises in the usual sense, but also on the tacit assumption that these premises constitute all the relevant information. It is amply clear that such inferences are common in ordinary discourse. It is also amply clear that such tacit assumptions of exhaustive relevance are not unavoidable. For instance, a typical tactic in solving run-of-the-mill recreational puzzles is to bring in information that is not contained in the given premises. However, there are no overwhelming reasons in evidence to show that the reasoning itself that is required for circumscriptive inference is different from our ordinary ways of reasoning. Hence it seems that we can simply pidgeonhole circumscriptive inference as a chapter in the theory of enthymemic reasoning, that is, in the theory of reasoning from partly tacit premises. And if so, circumscriptive inference can scarcely be claimed to be alternative to our normal logical inferences — or so it seems.

However, this is not the end of the story. The most important peculiarity of the tacit circumscriptive assumption is that it is not expressible in the language in which the inferences are carried out. This language is normally some familiar logical one not unlike a first-order language. However, the exhaustiveness of the given premises obviously cannot be expressed in such a language.

This shows both the interest of circumscriptive inference and the equivocal status of an attempt to formulate such inferences on a first-order level. The theoretically satisfactory approach would be to develop a logical language in which the tacit premise can be explicitly formulated. Only then could it be decided whether inferences from such premises require a logic different from our well-known modes of inference. It is also reasonable to expect that the logic of such richer languages would be an extension of, rather than an alternative to, the established logics

One can in fact view many other kinds of ampliative reasoning in the same way. This provides an interesting perspective on a number of "alternative logics". And this perspective is not restricted to recently developed "alternative logics".

The hidden assumptions on which applications of inductive logic can be argued to depend cannot be formulated in first-order languages even after simple probability measures are added to it. The theoretical situation hence seems to be the same as in the logic of circumscription, even though no one has tried to label inductive logic an "alternative" logic.

Similar situations occur also in mathematics. An interesting example is offered by maximality assumptions. Hilbert tried to make his axiomatization of geometry categorical by means of a maximality assumption that he called the axiom of completeness. ([Hilbert, 1899]; this axiom was actually introduced only in the second edition.) Gödel surmised that maximality assumptions not unlike Hilbert's axiom are what is needed in axiomatic set theory. Such assumptions are nevertheless hard to express in the usual logical languages like first-order languages. The reason is that the propositions of such a language work by imposing conditions on their models one model at the time, whereas maximality assumptions involve comparisons of different models with each other. This situation differs from the alternative logics in that Hilbert tried to formulate the additional axiom explicitly. However, such a formulation is impossible on the level on which most of the other reasoning is in Hilbert's book.

One can perhaps view first-order axiomatic set theory in the same light. The logic used there is precisely the traditional first-order logic. However, the characteristically set-theoretical modes of inference, such as the axiom of choice, cannot be adequately captured in traditional first-order logic. Hence the set theory incorporates only *Ersatz* versions of these principles of inference in the forms of axioms.

Why have logicians not developed richer languages in which such tacit assumptions could be expressed explicitly? Part of an answer was in the difficulty of such an enterprise. But perhaps part of the reason is that logicians have entertained too narrow a conception of what logic is. For one cannot help suspecting that somehow logicians have assumed that the familiar languages like first-order languages are capable of expressing all logical inferences. If so, we have here a telling example of the hegemony of traditional first-order logic as an answer to the title question of this article. Even logics that are calculated to be alternatives to the traditional first-order logic have been unwittingly colored by a belief in its unique role.

It nevertheless appears that the right project is to develop richer logical languages rather than devise different sets of special-interest modes of reasoning. If and when this is done, the result is likely to be a richer idea of what logic is – or of what it can be.

19 AMPLIATIVE INFERENCE AND ACQUISITION OF INFORMATION

In a different direction one can still ask for a unified perspective on ampliative inference. Such an overview on nondeductive inference must be connected with the notion of information, for the greatest common denominator of all ampliative inferences is that new information enters into an ongoing argument. A general theory of such inferences will have to focus on the source of the new information in comparison with alternative sources. For one thing, the reliability of the new information depends on the reliability of its source. Likewise, the evaluation of a new item of information depends on a comparison with the other items of information that might have been available instead of the actually introduced one.

One way of trying to reach such a general theory is to consider the reception of any new items of information as an answer to a (usually tacit) question. Indeed, if we know the source of an item of information and know what other items might be available from the same source or from others, etc., we conceptually speaking might as well consider such acquisition of the information as the reception of an answer to a question. From such a point of view, the theory of questions and answers assumes the role of a general framework for different kinds of ampliative inference. (See [Hintikka, 1999; Hintikka, Halonen and Mutanen, 1999].)

The theory of questions and answers is in turn based on epistemic logic, for a question is essentially a request of information [Hintikka, 1976; 2003b]. As was indicated in §3 above, a suitable interrogative logic can be developed as being determined largely as the logic of the declarative propositions that specify the epistemic state of affairs that a questioner asks to be brought about. Such a proposition is called the desideratum of the question in question. Needless to say, both the notion of a question and the idea of a source of answers must then be taken in a very general sense. The answerer need not be a human being or a database. It can be nature, one's environment, memory or even a questioner's imagination. Likewise, the act of asking a question may for instance take the form of an experiment or observation, or perhaps even a guess.

Such an "interrogative model of inquiry", as it has been called, is especially useful in the study of the strategic aspects of information seeking. For such a search amounts to a question-answer sequence interspersed naturally by deductive inferences. Strategies of inquiry will then amount to different methods of choosing the questions. The possibilities of theorizing opened by this perspective have not yet been used very widely. It is nevertheless clear that for instance some of the traditional "fallacies" are not breaches of the definitory rules of inference but violations of the reasonable principle of questioning, among them the so-called fallacy of petitio principii. (Cf. [Hamblin, 1970; Hintikka, 1987].)

Such interrogative inquiry is more closely related to the theory of the deductive inference than perhaps first meets the eye. This relationship in fact throws interesting light on the nature of logic in general. Formally, interrogative inquiry is partly analogous to deductive inquiry. In order to be in a position to ask a question, an inquirer must have established its presupposition. A step from the presupposition of a question to its answer (whenever available) is formally similar to a step from the premise (or premises) of a deductive inference to its conclusion. As a consequence, strategies of questioning govern the choices of presuppositions from the set of available propositions while strategies of deduction govern the choices of premises from the same pool of propositions.

In neither case can the optimal strategies be expressed in the form of recursive (mechanical) rules. However, there is a deep connection between the two kinds of strategies. In a context of pure discovery, that is, when all available answers are true and known to be true, the optimal strategies of interrogative inquiry coincide with the optimal strategies of deductions except for deviations caused by the possible unavailability of some answers. (Actually, another discrepancy may arise when the answers happen to deal with previously known entities.)

In other (somewhat looser) words, in a context of pure discovery interrogative inquiry is guided in effect by the same strategies as deduction. This result throws important light on what logic in general is, in particular what the place of deductive logic is in a wider schema of epistemological ideas. It suggests looking at deductive logic as a kind of systematic study *a priori* of what can happen in an actual empirical enterprise of information acquisition. It also goes a long way toward vindicating the old and still "Sherlock Holmes view" of logic as the gist of all good reasoning mentioned in §1 above. More generally speaking, it shows the relationship between deductive and ampliative reasoning.

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