



Cracow Logic Conference  
28 – 30 JUNE 2023

CONFERENCE BOOKLET

Jagiellonian University  
Kraków

	WEDNESDAY, 28 JUNE	THURSDAY, 29 JUNE	FRIDAY, 30 JUNE
9:00	REGISTRATION	TARMO UUSTALU	
9:30	WESLEY FUSSNER		JUDIT MADARÁSZ
10:00		COFFEE BREAK	
10:30	COFFEE BREAK	DANIEL GAINA	COFFEE BREAK
11:00	ANDRZEJ INDRZEJCZAK		PIOTR BŁASZCZYK
11:30		KATARINA MAKSIMOVIĆ	JANUSZ KACZMAREK
12:00	YAROSLAV PETRUKHIN	JACEK MALINOWSKI	MICHAŁ GIL-SANCHEZ
12:30			
13:00	LUNCH BREAK	LUNCH BREAK	LUNCH BREAK
13:30			
14:00			
14:30	YAROSLAV SHRAMKO	MICHAŁ BOTUR	EUGENIUSZ WOJCIECHOWSKI
15:00			ADAM OLSZEWSKI
15:30	MICHAŁ WROCŁAWSKI	KRZYSZTOF KRAWCZYK	KAZIMIERZ CZARNOTA
16:00	DARIUSZ KALOCIŃSKI	ATTILA JOÓ	
16:30	ZALÁN MOLNÁR	AMITAYU BANERJEE	
	INVITED	CONTRIBUTED	

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## 28 June 2023 - Day 1

### Interpolation in exchange-free logics

9:30 – 10:30

*Wesley Fussner*

*University of Bern*

In 1977, Maksimova proved a remarkable theorem: There are precisely seven consistent axiomatic extensions of intuitionistic propositional logic with the Craig interpolation property, and just three when negation is excluded from the language. In this talk, we explore what happens when some of the fundamental structural rules (exchange, weakening, contraction) of intuitionistic logic are dropped. In particular, we illustrate that for natural exchange-free variants of positive intuitionistic logic, there are continuum-many axiomatic extensions with the deductive interpolation property. Each of the aforementioned logics is characterized by linearly ordered and idempotent residuated lattice models, which need not be commutative due to the absence of the exchange rule. We further show that, in contrast, among logics characterized by linearly ordered and commutative idempotent algebraic models, just 60 have the deductive interpolation property. Our results illustrate that, at least in some contexts, the exchange rule presents a barrier to a logic having interpolation.

This is joint work with George Metcalfe and Simon Santschi.



### Proof-theoretic formulation of Quinean set theory NF

11:00 – 12:00

*Andrzej Indrzejczak*

*University of Łódź*

Term-forming operators, like  $\iota$ - or abstraction-operator, are very useful in practice but their theory is underdeveloped. It seems that providing well-behaved proof systems for such operators could help in the development of proof-theoretic approach to set theory. In the talk we present a cut-free sequent calculus for the general theory of term-forming operators, and two variants of its adjustment to abstraction-operators which allow us to formalise Quinean set theory NF.

### Normalization of Segerberg's natural deduction system for Boolean $n$ -ary connectives

12:00 – 12:30

*Yaroslav Petrukhin*

*University of Łódź*

In this talk, we will present a proof of the normalization theorem for Segerberg's natural deduction system for classical propositional logic being formulated in a language with at least one Boolean  $n$ -ary connective, which is a more general version of this theorem than usually considered.



### Dualities in constructive logics

14:30 – 15:30

*Yaroslav Shramko*

*Kryvyi Rih State Pedagogical University*

In my talk I will consider the phenomenon of logical duality with regard to some constructive logics. My approach will be mainly semantic - the dualization procedures will be based on some important properties of truth values used in constructive logics. The focus will be on an important logic of the constructivist family, constructive logic with strong negation, also known as Nelson's logic of constructible falsity. The logics in question will be equipped with suitable proof systems in the form of binary consequence systems, in particular such a system will be formulated for Nelson's explosive logic N3. It will be shown how this logic can be dualized semantically and syntactically, and how Nelson's logic and its dual can be combined in the general framework of a bi-Nelson logic.

## Degree spectra of relations on natural numbers and integers

15:30 – 16:00

*Michał Wroclawski*

*University of Warsaw*



Computable structure theory is an area of mathematics which combines computability theory with model theory. A structure is computable if both its domain and all functions and relations in its signature are uniformly computable. Computable structure theory asks the question: how does computational complexity of a structure vary between various isomorphic copies of that structure. Similarly, how does complexity of a computable relation on a structure vary between these copies.

Here we are going to be primarily concerned with the latter question. The intrinsic complexity of a relation will be measured by its degree spectrum, defined to be the set of all Turing degrees of isomorphic images of that relation in computable copies of a structure.

In [1] Wright considers the degree spectra of computable relations on standard ordering on natural numbers  $(\omega, <)$ . Using a certain type of priority construction he proves that except for some trivial functions, the degree spectrum of any computable relation on  $(\omega, <)$  contains all c.e. degrees. It is also easy to observe that it is contained in the set of all  $\Delta_2$  degrees.

In my presentation I will discuss my results obtained together with Dariusz Kalociński and Nikolay Bazhenov which are a continuation of Wright's research. I will show how a degree spectrum depends on various structural properties of different types of relations (especially functions). I will characterise a natural class of functions, called quasi-block functions, and show that all unary computable total functions outside of that class have a degree spectrum consisting of exactly c.e. degrees. I will provide a partial answer to the question what degree spectra of various quasi-block functions are and what they depend on. I also briefly mention the result that there are infinitely many possible degree spectra of unary computable functions on  $(\omega, <)$ .

In the final part of my presentation I am going to discuss how Wright's technique can be modified to prove analogous results for the ordering on integers. It turns out that in this case the proof becomes more complicated.

My and my coauthors' results are partly based on [2] and partly on currently unpublished research.

### References

- [1] Matthew Wright. Degrees of relations on ordinals. *Computability*, 7(4):349–365, 2018.
- [2] Nikolay Bazhenov, Dariusz Kalociński, and Michał Wroclawski. Intrinsic Complexity of Recursive Functions on Natural Numbers with Standard Order. In Petra Berenbrink and Benjamin Monmege, editors, 39th International Symposium on Theoretical Aspects of Computer Science (STACS 2022), volume 219 of *Leibniz International Proceedings in Informatics (LIPIcs)*, pages 8:1–8:20, Dagstuhl, Germany, 2022. Schloss Dagstuhl – Leibniz-Zentrum für Informatik.

## A hypothesis concerning intendedness of a model

16:00 – 16:30

*Dariusz Kalociński*

*Polish Academy of Sciences*



In the philosophy of mathematics, a distinction is being made between algebraic and non-algebraic disciplines. Non-algebraic disciplines, such as arithmetic, possess an intended structure, as exemplified by the standard model of arithmetic  $\mathcal{N} = (\mathbb{N}, +, \times, S, <, 0, 1)$ . If a first-order theory has non-intended models, the question arises of how to determine the intended one. I will start by reviewing main responses to this question, including second-order structuralism [1] and computational structuralism [2]. Next, I will propose a new hypothesis for capturing intendedness of a model. My account will be based on the methodological principle of parsimony and the explication of simplicity of a theory/model in terms of Scott analysis [3] (e.g., as Scott rank) which provides us with a well-behaved and robust hierarchy of complexity [4]. The proposed hypothesis states that if a theory has an intended (countable) model then it is the least complex model according to the assumed explication of simplicity. Initial confirmation of this hypothesis is provided by recent results on Scott analysis of non-standard models of PA [5]. I will try to give some preliminary philosophical evaluation of this proposal.

## References

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- [2] V. Halbach and L. Horsten, “Computational Structuralism,” *Philosophia Mathematica*, vol. 13, no. 2, pp. 174–186, 2005.
- [3] D. Scott, “Logic with Denumerably Long Formulas and Finite Strings of Quantifiers,” *Journal of Symbolic Logic*, vol. 36, no. 1, pp. 1104–329, 1965.
- [4] A. Montalbán, “A robust Scott rank,” *Proceedings of the American Mathematical Society*, vol. 143, no. 12, pp. 5427–5436, 2015.
- [5] A. Montalbán and D. Rossegger, “The structural complexity of models of arithmetic,” 2022. Preprint: <https://arxiv.org/abs/2208.01697>.

## Remarks on the ultrafilter extensions with infinite degree

16:30 – 17:00

*Zalán Molnár*

*Eötvös Loránd University*

The various connections between first-order and modal logic has always been a central issue in the model theory of modal logics. One of the central notions of these investigations is the concept of ultrafilter extension. The construction has clear analogies with ultrapowers when proving certain results in modal logic (cf. [5, 3]), hence the motto: “ultrafilter extensions in model theory of modal logics play the role similar to ultrapowers in first-order model theory”.

Although the two constructions are quite different from the perspective of first-order logic, in this talk we would like to stretch the analogy and see how far the similarities can go. It is quite natural to ask how the modal and first-order properties of a structure are connected to its ultrafilter extension. In general, the preservation of first-order sentences is  $\Pi_1^1$ -hard (cf. [2]), hence instead of formulas, we try to isolate classes for which the ultrafilter extensions and ultrapowers have interesting common features. In the recent work of [4] we showed that structures of bounded degree are both modally and elementary equivalent to their ultrafilter extensions. Below we present some follow up results, some of them are given without proofs.



## References

- [1] P. Blackburn, M. de Rijke, Y. Venema (2001). *Modal Logic*. Cambridge University Press.
- [2] Z. Molnár (2022). Ultrafilter extension of bounded graphs are elementary. Submitted to *Studia Logica*.

## 29 June 2023 - Day 2

### The proof theory of skew logics

*Tarmo Uustalu*  
*Reykjavik University*

9:00 – 10:00

I will talk about skew logics. By skew logics I mean substructural logics defined by categories with various types of skewstructure: skew monoidal categories and variations like skew closed or prounital closed categories, skew monoidal closed categories, partially normal skew monoidal categories, symmetric skew monoidal categories etc.

Skew monoidal categories (due to K. Szlachanyi) differ from normal (= ordinary) monoidal categories in that the structural laws of unitality and associativity of the tensor are not required to be natural isomorphisms; they are only natural transformations in a specific direction. Skew closed categories are a similar relaxation of closed categories. Partially normal skew monoidal categories are between skew and normal: they have some structural laws invertible, e.g., in left-normal skew monoidal categories, the left unitality law is a natural isomorphism.

In a series of papers, I, Niccolò Veltri, Cheng-Syuan Wan and Noam Zeilberger have studied skew logics using a particular proof-theoretical methodology.

For each logic we have considered, we have devised a sequent calculus whose derivations represent maps of the free category with the relevant skew structure. Furthermore, we have identified a subcalculus of normal-form derivations oriented at root-first proof search, with the property that every map is represented by exactly one derivation. This has given us for each logic a method to decide equality of two maps and a method to enumerate without duplicates all maps between a pair of objects (this is a finite set).

We have found that, because they are so weak (generally dropping commutativity and some directions of unitality and associativity), skew logics provide excellent insights into the fine anatomy of substructural logics. In particular, they explain which ingredients contribute to which characteristics of the stronger more customary substructural logics.

We started our project with purely multiplicative logics, but very recently Niccolò Veltri and Cheng-Syuan Wan analyzed a first combination of multiplicative connectives with additives. We now plan to address also modalities.

### Horn clauses in hybrid-dynamic quantum logic

*Daniel Gaina*  
*Kyushu University*

10:30 – 11:30

We present a simple yet expressive hybrid-dynamic logic for describing quantum programs. This version of quantum logic can express quantum measurements and unitary evolutions of states in a natural way based on concepts advanced in (hybrid and dynamic) modal logics. We then study Horn clauses in the hybrid-dynamic quantum logic proposed, and develop a series of results that lead to an initial semantics theorem for sets of clauses that are satisfiable.

### Lambda terms as graphs

*Katarina Maksimović*  
*University of Belgrade*

11:30 – 12:00

In this paper we are going to show how lambda terms can be presented with graphs. We will introduce the concept of a beta reduction graph first defined by Barendregt and we will investigate the connection between the normalization of lambda terms and beta reduction graphs. We will also show how certain relation on these graphs can enable us to define new equalities of lambda terms. The results presented in this paper are based on my PhD thesis titled Intensionality and the notion of algorithm where I investigated the problem of algorithm identity from a mathematical as well as philosophical perspective.



## Connexive Logics via Relating Semantics

12:00 – 12:30

*Jacek Malinowski*

*Polish Academy of Sciences*

There is a common agreement that each connexive logic should satisfy the Aristotle's and Boethian Theses (AB). However, the sole AB theses don't guarantee any common content or other form of "connexions" as they are true in binary matrix  $\{1, 0\}$  with distinguished value of 1, with classical material implication and negation defined as  $\neg 1 = \neg 0 = 1$ . Similarly, AB are true in a binary matrix with classical negation and implication defined as  $x \Rightarrow y = 1$  iff  $x = y$ .

It show that the sole AB theses are very weak and should be strengthen in some way. We can eliminate first counterexample by assuming that negation behaves in a classical way. It brings us to the notion of Boolean connexive logic. By a minimal Boolean connexive logic we mean the least set of sentences containing all classical tautologies expressed by means of  $\neg, \vee, \wedge, (A1), (A2), (B1), (B2), (A \rightarrow B) \supset (A \supset B)$  and closed under substitutions and modus ponens with respect to  $\supset$ .  $\supset$  denotes material implication.

In [1] we characterized Boolean connexive logics by means of relating semantics. Then Mateusz Klonowski proved that the class JT determines minimal Boolean connexive logics. The class JT  $\neg$  determines the least Boolean connexive logics satisfying the following two axioms:  $(A \rightarrow B) \supset (\neg\neg A \rightarrow \neg\neg B)$ ,  $(A \rightarrow B) \supset ((\neg A \rightarrow \neg B) \vee (\neg A \wedge B))$ .

Malinowski and Arturo Nicolas Francisco in [2] analyzed a number of properties added to AB in terms of relating semantics for Boolean connexive logics. In particular we show that Minimal Boolean Connexive Logic (or alternatively the logic determined by JT) is Abelardian, strongly consistent, Kapsner strong and antiparadox. We also construct examples showing that it is not simplificative, neither conjunction-idempotent nor strongly inconsistent logics.

## References

- [1] Jarmużek, Tomasz, and Jacek Malinowski 2019. Boolean Connexive Logics: Semantics and tableau approach. *Logic and Logical Philosophy* 28(3): 427–448.
- [2] Malinowski, Jacek and Ricardo Arturo Nicolas Francisco. Relating semantics for hyperconnexive and totally connexive logics, *Logic and Logical Philosophy*, forthcoming

## Tense algebras

14:30 – 15:30

*Michał Botur*

*Palacký University in Olomouc*

The aim of this talk is to present the development of algebras that model "tense" using operators, first on Boolean algebras, then on more complex structures that model some non-classical logics. We will show that the theory has seen significant development and originally logic theory can now be studied on general mathematical structures such as quantals.

## Congruence Extension Property in Varieties of Modal Algebras

15:30 – 16:00

*Krzysztof Krawczyk*

*Jagiellonian University*

In the talk I will present the theorem stating that there are uncountably many varieties of Boolean algebras with a single unary operator which lack the congruence extension property. The result provides a strong logical corollary about the lack of local deduction detachment theorem in congruential modal logics. It also remains in a sharp contrast with what is known about the normal modal logics. Thus, the presentation will show a drastic difference between the two families of logics.



## Weakly reflecting graph properties

16:00 – 16:30

Attila Joó

University of Hamburg

L. Soukup formulated an abstract framework in his introductory paper for proving theorems about uncountable graphs by subdividing them by an increasing continuous chain of elementary submodels. The applicability of this method relies on the preservation of a certain property (that varies from problem to problem) by the subgraphs obtained by subdividing the graph by an elementary submodel. He calls the properties that are preserved “well-reflecting”. The aim of this talk is to investigate the possibility of weakening of the assumption “well-reflecting” in L. Soukup’s framework. Our motivation is to gain better understanding about a class of problems in infinite graph theory where a weaker form of well-reflection naturally occurs.



## Interrelations between Kurepa’s principle, infinite Ramsey’s theorem, and a variant of Erdős–Dushnik–Miller theorem without the Axiom of Choice

16:30 – 17:00

Amitayu Banerjee

Alfréd Rényi Institute of Mathematics

**Theories:** ZFC (Zermelo–Fraenkel set theory with the Axiom of Choice (AC)), ZF (Zermelo–Fraenkel set theory without AC), ZFA (ZF with the Axiom of Extensionality weakened to allow the existence of atoms).

**Known informations:**

- In 1941, Ben Dushnik and Miller established the proposition “Every infinite graph contains either a countably infinite independent set or a clique with the same cardinality as the whole graph” in ZFC, and gave credit to Paul Erdős for the proof of the result for the case in which the cardinality of the graph is a singular cardinal. The above result is uniformly known as Erdős–Dushnik–Miller theorem.
- In 1958, Kurepa explicitly proved the proposition “Any partially ordered set such that all of its antichains are finite and all of its chains are countable is countable” (we abbreviate by K) in ZFC in response to a question raised by Sierpinski.
- Consider the following variant (abbreviated as EDM): “Every uncountable graph contains either a countably infinite independent set or an uncountable clique”. It is well-known that in ZFC, EDM implies K as well as the infinite Ramsey’s theorem (“Every infinite graph has either an infinite independent set or an infinite clique”).
- In 1969, Kleinberg proved that the infinite Ramsey’s theorem is not a theorem of ZF.
- In 1977, Andreas Blass studied the exact placement of the infinite Ramsey’s theorem in the hierarchy of weak forms of AC. In particular, he proved that the Boolean Prime Ideal Theorem (a weak form of AC) is independent of the infinite Ramsey’s theorem in ZF (i.e., there exists a ZF model where the Boolean Prime Ideal Theorem holds, but the infinite Ramsey’s theorem fails, and there exists a ZF model where the infinite Ramsey’s theorem holds, but the Boolean Prime Ideal Theorem fails) (see: <https://doi.org/10.2307/2272866>).
- In 2021, I studied some relations of K with weak forms of AC. (see: <https://doi.org/10.48550/arXiv.2009.05368>; to appear in *Commentationes Mathematicae Universitatis Carolinae*).
- In 2022, Eleftherios Tachtsis investigated the deductive strength of K without AC in more detail. Among several results, Tachtsis proved that  $DC_{\aleph_1}$  (Dependent Choices for  $\aleph_1$ , a weak form of AC stronger than Dependent Choices (DC)) implies K in ZF (see: <https://link.springer.com/article/10.1007/s00605-022-01751-9>).

**Talk on new results:** We study the exact placement of EDM in the hierarchy of weak forms of AC. In particular, we prove the following results (see <https://doi.org/10.48550/arXiv.2211.05665>, recently accepted in *Bulletin of Polish Academy of Sciences mathematics*):

1. The strength of EDM is strictly between  $DC_{\aleph_1}$  and K in ZFA.



2. EDM is strictly stronger than the infinite Ramsey's theorem in ZF (i.e., the infinite Ramsey's theorem does not imply EDM in ZF).
3. The Boolean Prime Ideal Theorem is independent of EDM in ZFA (specifically, neither the Boolean Prime Ideal Theorem implies EDM in ZF, nor EDM implies the Boolean Prime Ideal Theorem in ZFA).

## 30 June 2023 - Day 3

### Hajnal Andr  ka's conjecture on concept algebras of classical and relativistic spacetimes

9:30 – 10:30

*Judit Madar  sz*

*Alfr  d R  nyi Institute of Mathematics*

Hajnal Andr  ka considered two first-order models, one for classical spacetime and one for special relativistic spacetime. Her conjecture was that there is no concept algebra (cylindric algebra) between the concept algebras of these two spacetimes. We proved that this conjecture is true. This implies that relativistic spacetime (up to definitional equivalence) is the only proper reduct of classical spacetime that contains the concept of lightlike relatedness. In some sense, this means that there is no theory between special relativity theory and the kinematics of the late 19th century.

### Infinity of Euclid's straight line and circular inversion

11:00 – 11:30

*Piotr B  szczyk*

*Pedagogical University of Cracow*

1. Mathematics tamed the concept of infinity through numbers: Cantor's cardinal and ordinal numbers or inverses of infinitesimals developed by Euler. Sharing a similar understanding of finitude, be it a structure of natural numbers, or an Archimedean field, these two approaches diverge regarding infinity: Cantor ordinal arithmetic does not satisfy standard rules, e.g., commutativity, while inverses of infinitesimals comply with all the laws of an ordered field. The field of Conway numbers includes ordinal numbers and infinitesimals in one structure. Thus Euler's idea of infinity as an inverse of infinitesimals prevailed over Cantor's arithmetic of ordinal numbers [1],[2],[4]. We aim to implement the idea of infinity as the inverse of infinitesimals into Euclid's geometry.

2. The concept of infinity (apeiron) occurs in the definition of parallel lines and the Fifth Postulate, which evokes a line "being produced infinitely". Some view this proviso as potential infinity, meaning reiterated prolongation of a straight line [7]. We present a model of a semi-Euclidean plane to demonstrate that potential infinity does not guarantee straight lines satisfy the parallel axiom. It is a subspace of the Cartesian plane over the non-Archimedean field of hyperreal numbers in which angles in a triangle sum up to  $\pi$ , and the parallel axiom fails [3].

Standard models of non-Euclidean plane involve a non-Euclidean representation of straight lines (Poincare) or angles (Klein), in our model, both straight lines and angles are Euclidean. As all triangles in our model are also Euclidean, locally, it is the Euclidean plane, yet straight lines are 'too short' to meet the parallel postulate. We propose a characteristic of infinite straight lines in terms of Euclid's geometry alone with no reference to the concept of "being produced infinitely" or a number.

3. Since a semi-Euclidean and Archimedean plane satisfies the parallel postulate [6], what makes straight lines 'too short' are infinitesimal lines and angles. We introduce infinitesimals by negating Aristotle's axiom and thus do not refer to numbers [5]. Then we study inverses of infinitesimals.

The geometric counterpart of a multiplicative inverse operation in a field is the construction of circular inversion (Elements, III.37). We show that it guarantees straight lines meet the parallel postulate and present an equivalent version of that construction. It is that for any base and acute angle, there exists an isosceles triangle. It is an equivalent version of the Fifth Postulate showing inverses of infinitesimals exists in a plane.

#### References

- [1] B  szczyk, Petiurenko 2022. Euler's series for sine and cosine. An interpretation in nonstandard analysis.
- [2] B  szczyk 2021. Galileo's paradox and numerosities.
- [3] B  szczyk, Petiurenko 2021. Commentary to Book I of the Elements. Hartshorne and beyond.
- [4] B  szczyk, Fila 2020. Cantor on infinitesimals. Historical and modern perspective.
- [5] Greenberg 2007. Euclidean and Non-Euclidean Geometries.
- [6] Hartshorne 2000. Geometry: Euclid and Beyond.
- [7] Linnebo, Shapiro 2019. Actual and potential infinity.



**On the Negation of the Atomic State of Affairs.  
Algebraic and Topological Analysis (Interpretation)****11:30 – 12:00***Janusz Kaczmarek*  
*University of Łódź*

The problem I will address concerns Wittgenstein's ontology from his *Tractatus*. At the 2018 Wittgenstein Symposium in Kirchberg, Prof. Weingartner, after my talk (presentation), asked: what is the negation of the atomic state of affairs? This is an important problem from the point of view of Wittgenstein's ontology contained in the *Tractatus*. I will therefore show what the algebraic solution is (in the framework of lattice theory, including Wolniewicz's lattices) and what the solution is in general topology.

In the first case we obtain the theorem: 1) the negation of an atomic state of affairs is various elementary situations in Wolniewicz's sense (both the atomic situation and the complex situation, e.g. the possible world). In the second case (topological approach) it will turn out that: 2) no elementary situation is the negation of an atomic state of affairs.

Hence the conclusion is born: there are no 'negative states of affairs'; all states of affairs - contrary to Wittgenstein's terminology - are positive.

**Bibliography**

- [1] Kaczmarek, Janusz (2019) *Ontology in Tractatus Logico - Philosophicus. A Topological Approach*, [in:] G. M. Mras, P. Weingartner, B. Ritter (Eds.), *Philosophy of Logic and Mathematics, Proceedings of the 41st International Ludwig Wittgenstein Symposium*, De Gruyter, pp. 246–262
- [2] Wittgenstein, Ludwig (1921), *Logisch - philosophische Abhandlung*, [in:] *Annalen der Naturphilosophie* (translated by C. K. Ogden)
- [3] Wolniewicz, Bogusław (1999), *Logic and Metaphysics. Studies in Wittgenstein's Ontology of Facts*, *Biblioteka Myśli Semiotycznej*, no. 45, (ed.) J. Pelc, Warsaw

**Invariants preserved by the information projection****12:00 – 12:30***Michał Gil-Sanchez*  
*Jagiellonian University*

Probability kinematics offers a means to update one's beliefs regarding a specific piece of evidence involving a partition of the space of potential outcomes, so called Jeffrey Conditionalisation. This updating process guarantees that the relative proportions between possibilities are preserved within each partition cell. Furthermore, Jeffrey Conditionalisation can be seen as a specific instance of Information-Projection, and analogous updating invariants can be identified when the evidence is expressed as a set of linear equations. As a result, these new invariants can be referred to as Generalised Proportions, with a classic example being the odds ratio.

**Intuicyjna interpretacja logiki trójwartościowej i logiki czterowartościowej****14:30 – 15:00***Eugeniusz Wojciechowski*  
*Poland*

Jerzy Śłupecki zbudował pewną konstrukcję logiczną, która jego zdaniem jest milcząco zakładana przez Jana Łukasiewicza w jego logice trójwartościowej. Konstrukcja Śłupeckiego została poddana krytyce przez Ludwika Borkowskiego. Uwzględnia on w niej również uwagi krytyczne jakie pojawiły się na zjeździe w Zürichu w 1938, podczas dyskusji nad referatem Łukasiewicza *Die Logik und das Grundlagenproblem*. Zakwestionowano tam intuicje związane z implikacją trójwartościową. Najpoważniejszy argument, niezgodności z zakładanymi tu intuicjami, został sformułowany później przez A.N. Priora - koniunkcja dwóch zdań o wartości  $1/2$  ma również wartość  $1/2$ , co pociąga za sobą to, że koniunkcja zdania o wartości  $1/2$  z jego negacją ma ją również. Rezultat krytycznej analizy Borkowskiego doprowadził go do modyfikacji i rozszerzenia konstrukcji Śłupeckiego, w kierunku zbudowania podstaw pod logikę czterowartościową, zgodną z propozycjami logiki Łukasiewicza tego

typu. Nie referujemy tu bliżej konstrukcji Śłupeckiego jak i konstrukcji Borkowskiego, ponieważ proponujemy inną bazę interpretacyjną. Pozwoli ona na interpretację zarówno logiki trójwartościowej jak i czterowartościowej.

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## About logically probable sentences

15:00 – 15:30

*Adam Olszewski*

*Pontifical University of John Paul II in Cracow*

The starting point of this paper is the empirically determined ability to reason in natural language by employing probable sentences. A sentence is understood to be logically probable if its schema, expressed as a formula in the language of classical propositional calculus, takes the logical value of truth for the majority of Boolean valuations, i.e., as a logically probable formula. Then, the formal system P is developed to encode the set of these logically probable formulas. Based on natural semantics, a strong completeness theorem for P is proved. Alternative notions of consequence for logically probable sentences are also considered.

## Leśniewski's mereology as the partition theory of real objects and intentional objects (according to Ingarden's concept)

15:30 – 16:00

*Kazimierz Czarnota*

*Polskie Towarzystwo Filozoficzne*

I introduced the theoretical principles of the mereological division of time and space at the meeting of the Polish Philosophical Society on October 31, 2022, where I presented a paper entitled: Non-Standard Theory of Collective sets (pointless time and space).

In this presentation I will be using the following terminology:

- Proper part = part. I do not use an expression 'improper part'

## FORMAL ONTOLOGY. PREMISES

A physical object (material object, thing) has the following characteristics:

- it is space-time — understood as a three-dimensional matter lasting in time.  
Neither a point, a line, a plane, nor a time section are material objects. Instead, they are abstract objects.
- it is partially isolated: A fully isolated object is an abstract entity.
- it has mass and energy, whose value changes when exchanging with environment.



- it has specific features, such as hardness, fluidity, temperature, color
- it remains in relations with other physical objects, e.g. via direct contact, or via electromagnetic field
- it has a limited size and time of existence, it emerges and ends.

#### FORMAL DESCRIPTION OF THE PHYSICAL PARTITION AND EXAMPLES

**Every part of a thing is a thing.**

#### THE THEORETICAL PARTITION

1. The questions of cosmic, macroscopic and microphysical systems in the context of partition.
2. Homogeneous material partition
3. The existence of indivisible parts – “atoms”

Cracow Logic Conference (CLOcK) is the oldest Polish conference series on logic. For many years it existed under a deceptive name Konferencja Historii Logiki (Conference on the History of Logic). The present conference is 68th in the series. It will be held on 28-30 June 2023, at Department of Logic, Institute of Philosophy, Jagiellonian University in Kraków.

**List of topics include**

- Algebraic logic
- Model theory
- Proof theory
- Philosophical logic
- History of logic

**Program and Organising Committee:** Zalan Gyenis, Tomasz Kowalski, Piotr Łukowski, Katarzyna Słomczyńska, Adam Trybus

**Venue:** The conference will be held in the Institute of Philosophy building on Grodzka 52, room 28.

**Web-page:** <https://iphils.uj.edu.pl/~a.trybus/clock/>

**Contact:** Email us at `dlg` at `iphils.edu.pl`

