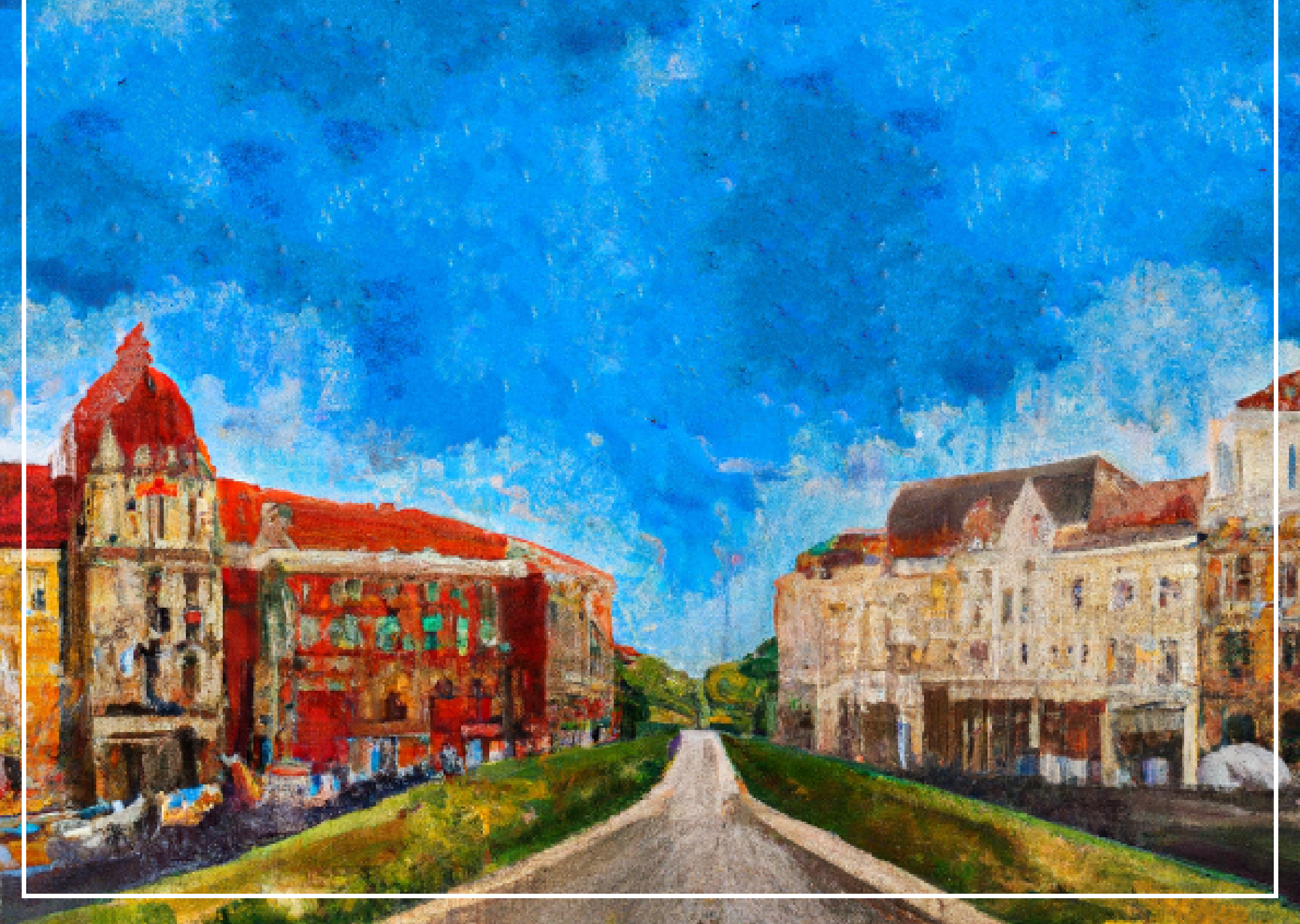
Cracow Logic Conference

CLoCk

Trends in Logic XXIV

Kraków, 18-21 June, 2024



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9:30	Jacek Malinowski			
10:00	Anna Brożek, Zofia Hałęza	COFFEE BREAK	COFFEE BREAK	COFFEE BREAK
10:30		Mai Gehrke	Marta Bílková	Ondrej Majer Camillo Fiore
11:00	Yde Venema			Ondrej Majer Camillo Fiore
11:30		Wesley Fussner Piotr Kulicki	Javier Vineta Paweł Płaczek	Piotr Błaszczyk Andrea Sabatini
12:00	LUNCH BREAK	LUNCH BREAK	LUNCH BREAK	LUNCH BREAK
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16:30				

18:00 DINNER

INVITED

CONTRIBUTED

About

Cracow Logic Conference (CLoCk) is the oldest Polish conference series on logic. For many years it existed under a deceptive name Konferencja Historii Logiki (Conference on the History of Logic), and was for the most part limited to the Polish logic community. This year's conference will be the 69th in that series: LXIX KHL for those who care. Since 2023, CLoCk went truly international and welcomes contributions on all areas of symbolic logic. Purely philosophical contributions are not automatically excluded, but are frowned upon. CLoCk also sees itself as a continuation of the AsubL workshop series, so mathematical approaches to nonclassical logics are preferred.

Trends in Logic is the conference series of the journal Studia Logica aimed at worldwide promotion of logic and Studia Logica. The series began in 2003, and have been held annually at different logic centres. Apart from Poland, Trends in Logic conferences were held in Denmark, China, Belgium, The Netherlands, Germany, Italy, USA, Georgia, Argentina, Brazil, Russia and Ukraine. The series has been instrumental in increasing the visibility of Studia Logica and elevating its international standing.

The conference is hosted by the Department of Logic, Institute of Philosophy, Jagiellonian University in Kraków, Co-organized by the JU Doctoral School in the Humanities.

Invited speakers

- Guillermo Badia
- Marta Bílková
- Thomas Ferguson
- Mai Gehrke
- Dorota Leszczyńska-Jasion
- Francesco Paoli
- Michał Stronkowski
- Kordula Świętorzecka
- Yde Venema

Program committee

Andrzej Indrzejczak, Tomasz Kowalski, Piotr Łukowski, Jacek Malinowski, Heinrich Wansing

Organizing committee

Zalán Gyenis, Tomasz Kowalski, Krzysztof Krawczyk, Piotr Łukowski, Adam Trybus

Timetable

Tuesday, June 18, 2024

9:00-9:30	**Registration, opening**	
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10:00-10:30	**Anna Brożek[1] and Zofia Hałęza[2]** [1]Uniwersytet Warszawski, [2]Uniwersytet Łódzki	The Origin of Studia Logica and Warsaw as the World Capital of Mathematical Logic
10:30-11:00		General discussion of the history of Studia Logica
11:00-12:00	**Yde Venema** Institute for Logic, Language and Computation	Propositional Dynamic Logic (re)visited
12:00-14:00	**Lunch break**	
14:00-15:00	**Michał Stronkowski** Warsaw University of Technology	Profinite Heyting algebras
15:00-15:30	**Break**	
15:30-16:00	**Thomas Ferguson** Rensselaer Polytechnic Institute	Bounds Consequence and Liberalizing Semantic Values

Wednesday, June 19, 2024

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10:30-11:30		**Mai Gehrke** Université Cote d'Azur	Canonical extension of lattices
11:30-12:00	A	**Wesley Fussner** Institute of Computer Science of the Czech Academy of Sciences	Poset Products and Strict Implication
	B	**Piotr Kulicki** The John Paul II Catholic Univertsity of Lublin	On norms defined on sequentially composed actions
12:00-14:00	**Lunch break**		
14:00-14:30	A	**Michał Wrocławski** University of Warsaw, Faculty of Philosophy	Punctual presentability of injective structures and trees
	B	**Sebastian G.W. Speitel** University of Bonn	Logicality and Determinacy
14:30-15:00	A	**Katarzyna Słomczyńska** University of the National Education Commission, Kraków	Free p-algebras
	B	**Agata Tomczyk** Adam Mickiewicz University	Sequent Calculi for Two non-Fregean Theories
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15:30-16:00	A	**Filip Jankovec** Institute of Computer Science, CAS	Infinitary semilinear extensions of Abelian logic
	B	**András Kornai** Budapest University of Technology	Mechanical causation as relevant implication
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11:30-12:00	A	**Javier Vineta** Departamento de Filosofia de la Universidad de Navarra	On a Generalization of all Strong Kleene Generalizations of Classical Logic
	B	**Paweł Płaczek** WSB Merito University in Poznań, Poland	Semiassociative Lambek Calculus: Sequent systems and algebras
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	B	**Ren-June Wang** Department of Philosophy, National Chung-Cheng University	On the normal form of deductions in sequent calculus for intuitionistic logic

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	B	**Camillo Fiore** University of Buenos Aires / IIF-SADAF-CONICET	Notational Variance in Substructural Logics
11:30-12:00	A	**Piotr Błaszczyk** University of the National Education Commission, Krakow, Poland	Reading Newton's De Analysi by hyperfinite sums
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	B	**Agustina Borzi** IIF-SADAF-CONICET, UBA	General Tableaux Method for Metainferential Logics
14:30-15:00	A	**José Martín Castro-Manzano** UPAEP University	Non-Deductive Term Logic Tableaux
	B	**Rodrigo Mena Gonzalez** Ludwig-Maximilians-Universität München	One Problem from Carnap and Wójcicki
15:00-15:30	A	**Nicolò Zamperlin** University of Cagliari	Generalized set-assignment semantics for Parry systems
	B		

Abstracts

Tuesday 18th

Studia Logica. Past, present and future

Jacek Malinowski

9:30–10:00

Studia Logica, and Department of Logic and Cognitive Science, Polish Academy of Science

This is a presentation opening Studia Logica conferences Trends in Logic. It presents journal missions, editors, editorial board, journal special issues, Trends in Logic conferences, Studia Logica Library book series consisting of Trends in Logic, Outstanding Contributions to Logic and logic in Asia, brief journal history and scientometrics.

The Origin of Studia Logica and Warsaw as the World Capital of Mathematical Logic

Anna Brożek[1], ***Zofia Hałęza***[2]

10:00–10:30

[1]Uniwersytet Warszawski,
[2]Uniwersytet Łódzki

In 1929, Jan Łukasiewicz published an article titled "O znaczeniu i potrzebach logiki matematycznej" [On the Significance and Needs of Mathematical Logic], where he mentioned establishing a specialized logic journal among the most urgent needs of contemporary Polish logic. Łukasiewicz was aware of the significant role that the journals Przegląd Filozoficzny [Philosophical Review] (founded in 1897) and Ruch Filozoficzny [Philosophical Movement] (founded in 1911) had played in the development of Polish philosophy, as well as the role that Fundamenta Mathematicae (founded in 1920), had begun to play in the world mathematics. He believed that the logical milieu that had already been established in Warsaw, was ready to issue an international journal that would provide the possibility of exchanging the results and new impulses for further developments in this domain. Thus, it is known that since the late 1920s, Łukasiewicz and his collaborators had been making efforts to establish a purely logical periodicals. These ideas took two forms. In 1934, the first volume of Studia Logica was published – conceived as a series of monographs written by collaborators of the Łukasiewicz's Seminar. Three years later, preparations began for publishing a regular multilingual journal, Collectanea Logica. The first volume of this journal was already ready in the printing house at the end of August 1939. Unfortunately, the entire edition burned down during the bombing of Warsaw by the Luftwaffe. Only copies of individual articles survived, handed over to the authors. The idea of founding a purely logical journal was revived by Kazimierz Ajdukiewicz only in 1953, nearly twenty years after Łukasiewicz's first trials. Ajdukiewicz gave it the title "Studia Logica"; the journal referred in its title to the title of Łukasiewicz's series of monographs, but in its content to Łukasiewicz's Collectanea Logica. In the presentation, we will discuss the fate of the first issue of the journal Studia Logica against the backdrop of the richness of Polish scientific life in the interwar period, when Warsaw, with its two chairs of mathematical logic and a genuine school logic, could rightly be called the World's Capital of Logic.

Propositional Dynamic Logic (re)visited

Yde Venema

11:00–12:00

Institute for Logic, Language and Computation

Propositional dynamic logic (PDL) is a well-known modal logic stemming from the wave of so-called process logics that emerged in the 1970s. Characteristic to PDL is that its collection of modalities is given as the set of regular expressions over some set of atomic programs and (possibly) so-called test program. The main results on PDL, such as a sound and complete axiomatisation and the decidability and computational complexity of its satisfiability problem, were obtained relatively soon after its introducton. In the talk I will review some results on PDL that have been obtained or re-evaluated in the past decade. Topics to be discussed include its relation with other fixpoint logics, expressive completeness results, cut-free proof systems, and interpolation properties.

Profinite Heyting algebras

***Guram Bezhanishvili*[1], *Nick Bezhanishvili*[2], *Tommaso Moraschini*[3], *Michał Stronkowski*[4]**

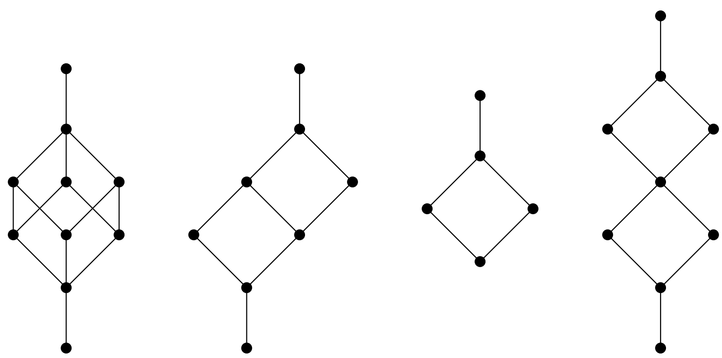
14:00–15:00

[1]Department of Mathematical Sciences, New Mexico State University
[2]Institute for Logic, Language and Computation, University of Amsterdam
[3]Department of Philosophy, University of Barcelona
[4]Faculty of Mathematics and Information Science, Warsaw University of Technology

Profinite structures appear in various parts of mathematics and computer science, e.g., in Galois theory, algebraic number theory, and formal language theory. We investigate profinite algebras in the context of algebraic logic, to be more precise, profinite Heyting algebras. Let us recall that intermediate (propositional) logics are logics placed between classical and intuitionistic logics. They are in one-to-one correspondence with nontrivial varieties of Heyting algebras. A corresponding variety of Heyting algebras constitutes an algebraic semantics for a given intermediate logic.

An algebra is profinite if it is an inverse limit of an inverse system of finite algebras. In particular, it is a topological algebra and its topology is compact, Hausdorff, and totally disconnected. An important case appears when a profinite algebra $\widehat{\mathbf{A}}$ is an inverse system of finite homomorphic images of a particular algebra $\mathbf{A}$. In such a case, we say that $\widehat{\mathbf{A}}$ is a profinite completion of $\mathbf{A}$. There are many well-known examples of profinite algebras which are not profinite completions. However, finding such a Heyting algebra occurred to be a challenging problem posed by Bezhanishvili and Morandi in 2009 [2]. We present a solution to this problem in the following strong form.

Theorem 1 ([1]). *Let $\mathcal{V}$ be a variety of Heyting algebras. The profinite members of $\mathcal{V}$ are profinite completions if and only if the Heyting algebras depicted below do not belong to $\mathcal{V}$.*



The main tool in the proof is Esakia duality between Heyting algebras and Esakia spaces. This allow us to present a connection of our work with Esakia's representation problem: which ordered sets are order-reducts of Esakia spaces. (This is a variant of the classical Grätzer's and Kaplansky's problem which asks, in modern terminology, which ordered sets are order-reducts of Priestley spaces.)

References.

[1] Guram Bezhanishvili, Nick Bezhanishvili, Tommaso Moraschini, and Michał Stronkowski. Profiniteness and representability of spectra of Heyting algebras. *Adv. Math.*, 391:47, 2021. Id/No 107959.

[2] Guram Bezhanishvili and Patrick J. Morandi. Profinite Heyting algebras and profinite completions of Heyting algebras. *Georgian Math. J.*, 16(1):29–47, 2009.

Bounds Consequence and Liberalizing Semantic Values

Thomas Ferguson

15:30–16:30

Rensselaer Polytechnic Institute

Bounds consequence (introduced by Greg Restall and defended by David Ripley) provides a reading of logical consequence in which a sequent is understood as a position in which an agent accepts (or asserts) all formulae in the antecedent and rejects (or denies) all formulae in the succedent. Provability of a sequent is then understood as a certificate that for an agent to take that position has landed "out-of-bounds" in some sense. Its introduction has provided an opportunity to recast a number of a priori pragmatic phenomena governing our conversational norms as authentically semantic, including features like topicality, confidentiality, and justification. In this talk I will present work (joint with Jitka Kadlecikova) in which even conversational aspects like concern for one's interlocutor can be understood as guiding the determination of these bounds and can receive a semantic representation in model theory.

Wednesday 19th

A modular bisimulation characterisation for fragments of hybrid logic

G. Badia[1], ***D. Gaina***[2], ***A. Knapp***[3], ***T. Kowalski***[4], ***M. Wirsing***[5]

9:00–10:00

[1]U. of Queensland
[2]Kyushu U.
[3]U. of Augsburg
[4]UJ
[5]U. Munich

There are known characterisations of several fragments of hybrid logic by means of invariance under bisimulations of some kind. The fragments include $\{\downarrow, @\}$ with or without nominals (Areces, Blackburn, Marx), @ with or without nominals (ten Cate), and $\downarrow$ without nominals (Hodkinson, Tahiri). Some pairs of these characterisations, however, are incompatible with one another. For other fragments of hybrid logic no such characterisations were known so far. We prove a generic bisimulation characterisation theorem for all standard fragments of hybrid logic, in particular for the case with $\downarrow$ and nominals, left open by Hodkinson and Tahiri. Our characterisation is built on a common base and for each feature extension adds a specific condition, so it is modular in an engineering sense.

Canonical extension of lattices

Mai Gehrke

Université Cote d'Azur

10:30–11:30

Canonical or perfect extensions of ordered algebras originate with the seminal work of Jónsson and Tarski on Boolean Algebras with Operators published in 1951-52. Since then the notion has been generalized to encompass arbitrary additional operations and more general underlying order structures. Canonical extensions are closely related to topological dualities for these algebraic structures and provide a powerful abstract setting in which to study duality and relational semantics. This talk provides a survey, mainly focused on the setting of bounded lattices with additional operations, and reflects the content of an upcoming research monograph co-authored with Wesley Fussner.

Poset Products and Strict Implication

Wesley Fussner[1], ***Peter Jipsen***[2]

11:30–12:00

[1]Institute of Computer Science of the Czech Academy of Sciences
[2]Chapman University

Poset products were introduced by Jipsen and Montagna in order to give representation theorems for several classes of residuated lattices, notably GBL-algebras. In this work, we discuss poset products as a formalism for realizing various substructural logics as logics whose implication connective is a strict implication. Our work rests on new results on which residuated lattices may be isomorphically respresented as poset products of appropriately chosen algebras. In particular, we generalize the well-known result that states that Heyting algebras of up-sets (of some poset) are exactly the perfect Heyting algebras.

On norms defined on sequentially composed actions

Piotr Kulicki

11:30–12:00

John Paul II Catholic University of Lublin

This work provides a formal exposition of sequentially composed actions, enhancing deontic action logic with novel perspectives and complexities. It is grounded in an algebraic framework that accounts for both successful and failed enactments of actions, as initially presented in [4]. On this basis, we delineate two separate frameworks of obligation concerning sequentially composed actions. The first framework adheres to local norms that govern one-step actions, reflecting the model discussed in [4]. The second framework evaluates the results of entire action sequences in the context of an agent's objectives, paralleling the methodology in [1] and inspired by Andersonian-Kangerian reduction strategies, exemplified in [7]. This exploration broadens previous research on the deontic properties of actions and states, as explored in [5, 6], specifically focusing on sequentially composed actions. Local norms underpin action-based obligations, while goal-oriented norms align with state-based obligations.

For norms based on local contexts, we apply the Standard Deontic Logic model for clarity and brevity, as noted in [7]. Hence, norms that govern single-step actions are established based on prohibitions. An action is considered permissible if it is not explicitly prohibited, and obligatory if the failure to perform it is prohibited. On this premise, we define norms for sequentially composed actions as proposed in [4]. Essentially, a composed action is prohibited if any of its individual steps are locally prohibited, it is permissible if each step is individually permissible, and it is obligatory if every constituent step is obligatory and the sequence is defined in such a way that it can be successfully completed regardless the choices of its executor.

Regarding norms oriented towards goals, the primary concept is that in every situation an agent faces, there are optimal states the agent ought to aim for. Norms for sequentially composed actions are then derived based on whether these optimal states are achieved or not. We examine several ways of establishing obligations as preliminarly sketched in [2]. To complete the formal system we consider the interplay between local norms and goal-oriented norms.

In a recent paper [3] Ju & Nygren made another attempt to formalize the normative properties of sequential actions was made. The main idea there is to analyse current behaviour of an agent in the context of their past actions. As a conclusion of this presentation we will compare selected outcomes of our approach with those of Ju & Nygren's.

References.

[1] Janusz Czelakowski. Deontology of compound actions. *Studia Logica*, 108(1):5–47, 2020.

[2] Fengkui Ju and Piotr Kulicki. Actions and Deontology: Janusz Czelakowski on Actions and Their Assessment. In Jacek Malinowski and Rafał Palczewski, editors, *Janusz Czelakowski on Logical Consequence*, pages 265–286. Springer Verlag, 2024.

[3] Fengkui Ju and Karl Nygren. Normative properties of sequential actions. In Juliano Maranhão, Clayton Peterson, Christian Straßer, and Leendert van der Torre, editors, *Deontic Logic and Normative Systems - 16th International Conference, DEON 2023, Trois-Rivières, QC, Canada, July 5-7, 2023*, pages 139–157. College Publications, 2023.

[4] Piotr Kulicki and Robert Trypuz. Completely and partially executable sequences of actions in deontic context. *Synthese*, 192(4):1117–1138, 2015.

[5] Piotr Kulicki and Robert Trypuz. Connecting actions and states in deontic logic. *Studia Logica*, 105(5):915–942, 2017.

[6] Piotr Kulicki, Robert Trypuz, Robert Craven, and Marek J. Sergot. A unified logical framework for reasoning about deontic properties of actions and states. *Logic and Logical Philosophy*, 32(4):583–617, 2023.

[7] Paul McNamara and Frederik Van De Putte. Deontic logic. In Edward N. Zalta and Uri Nodelman, editors, *The Stanford Encyclopedia of Philosophy*. Metaphysics Research Lab, Stanford University, fall 2022 edition, 2022.

Punctual presentability of injective structures and trees

Michał Wrocławski

14:00–14:30

University of Warsaw, Faculty of Philosophy

This presentation is based on joint research with Nikolay Bazhenov, Ivan Georgiev, Dariusz Kaloci´nski, Luca San Mauro, and Stefan Vatev (in alphabetical order). A similar talk has been accepted for presentation at Logic Colloquium 2024. Together with D. Kaloci´nski and L. San Mauro we also intend to send a related article to the 49th International Symposium on Mathematical Foundations of Computer Science (to be presented at the conference and included in its proceedings).

Punctual structure theory considers structures whose domain is $\mathbb{N}$ and all relations and functions from their signature are uniformly primitive recursive.

In [1] it has been proved that every computable structure of each of the following classes has a punctual presentation: equivalence structures, linear orderings, Boolean algebras and some more. On the other hand, among others, computable torsion abelian groups do not always have a punctual presentation.

We have considered a similar question with respect to other types of structures, in particular to structures of the form $(\mathbb{N}, f)$ where f is a unary injective function (which we call injective structures). An injective function can be decomposed into various orbits, each of which is either a cycle, or constitutes an $\mathbb{N}$-chain or a $\mathbb{Z}$-chain. A function which can be decomposed into cycles only is called a cyclic function.

We have shown that every computable injective structure which is not cyclic is punctually presentable. So is every computable injective structure which has some cycle length occurring infinitely many times.

When none of these cases occurs, the situation is less clear. We have constructed a computable cyclic injective structure without a punctual presentation. We have also considered the following concept. Suppose that f is a cyclic function and that $g : \mathbb{N} \to \mathbb{N}$ is an enumeration of all cycle lengths of f in which each cycle length is repeated as many times as it appears in f. We have proved several charaterizations tying punctual presentability of $(\mathbb{N}, f)$ to computational properties of g.

We have also had success in proving some cases of a theorem which states a connection between instrinsic punctuality of an additional relation on a punctual injective structure and definability of that relation using a certain restricted class of quantifier-free formulas of first-order logic, assuming that a certain effectivness condition is satisfied. This is aimed to lead to a more general punctual version of Ash-Nerode Theorem.

In addition to injective structures, we have also considered trees interpreted as either (predecessor) functions or as relations. We have shown that while every computable tree as a relation has a punctual presentations, the situation is different when trees as functions are considered.

References.

[1] Iskander Kalimullin, Alexander Melnikov, Keng Meng Ng, Theoretical Computer Science, pp. 73–98, Algebraic structures computable without delay, volume 674, 2017.

Logicality and Determinacy

Sebastian G.W. Speitel

14:00–14:30

University of Bonn

According to Tarski's celebrated *model-theoretic definition of logical consequence* a sentence φ follows logically from a class of sentences Γ if and only if every model of the sentences in Γ is also a model of the sentence φ [12]. Explicating the notion of logical consequence for a wide range of logics, the definition relies on a prior division of the expressions of the language of φ and Γ into logical and extra-logical: the logical constants of a language directly influence the range of admissible models to take into account when assessing a claim of consequence and thereby determine which arguments are logically valid.

Since logical consequence is assumed to be a *formal* relation, i.e., one that is not influenced by extraneous 'empirical' information, the choice of which expressions to count as logical is not arbitrary: it must be such as to ensure that inferences licensed on the basis of that choice respect the constraint of formality. Determining the right boundary between logical and non-logical constants is thus of crucial importance to the Tarskian project of providing an adequate model-theoretic explication of the notion of logical consequence. If too many, or inappropriate, expressions are classified as logical several 'material', non-formal transitions will, wrongly, qualify as logical consequences; if too few expressions are taken to be logical the resulting relation of logical consequence will be impoverished and not yield an adequate account for a given language. This is the *demarcation problem of the logical constants.*

Although it is easy enough to provide a satisfactory division of expressions for common logical languages, such classifications often proceed by means of uninformative enumerations. This unprincipled and case-by-case determination contrasts sharply with the generality of the definition of logical consequence which applies to all languages of a particular type indiscriminately. Tarski himself regarded the project of providing a mathematically rigorous explication of the notion of logical consequence as unfinished until this crucial issue could be addressed. What is needed to put the model-theoretic definition of logical consequence on a firm philosophical foundation and shield it against skeptical attacks is, therefore, a *criterion of logicality*, a set of mathematically precise and philosophically informative principles which delineate, for the kinds of languages covered by Tarski's definition, the appropriate class of logical constants.

In the tradition of devising criteria to solve the demarcation problem of the logical constants *invariance-based approaches* hold a prominent place ([13], [11], [7], [1]). These criteria ground the formality of logical inferences in properties of the model-theoretic denotations of purported logical constants. Despite oftentimes regarded as a *necessary* component of delineating the logical expressions of a language, purely invariance-based criteria appear to face issues they are, by their very design, unable to overcome. This is due to the fact that criteria of this sort, for the most part, only address the *semantic question* of what constitutes a logical denotation while neglecting the attendant *meta-semantic question* of how logical constants come to denote such denotations. A theory of logical inference as *safe and reliable* transition between premisses and conclusions ought to address both questions, however.

In this talk, I will present, motivate and defend a criterion of logicality which supplements invariance-based constraints with inferentialist requirements on the determination of meaning and explore its scope and some of its consequences. Central to the proposed criterion is a combination of insights from two traditions in the philosophy of logic and language which, together, address both the semantic and meta-semantic question: from the model-theoretic tradition it adopts the idea that the formality of logical consequence is grounded in properties of the denotation of logical expressions, best captured by an *invariance constraint.* From the inferentialist tradition it takes up the insight that the meaning of a logical expression should be recoverable from its inferential behaviour, that its meaning should be *uniquely determined by its inferential role.* This is operationalized by means of a *categoricity-requirement.* The resulting criterion demands that for an expression to be logical the inferential and model-theoretic aspects of its meaning must cohere in such a way that its inferential behaviour (codified by a consequence relation) uniquely determines one among its consistent, formal – i.e., (isomorphism-)invariant, – model-theoretic values. The criterion was developed in joint work with D. Bonnay (see [2]).

The prospects of a combined criterion of the kind outlined above is seriously threatened by a set of underdetermination phenomena, sometimes referred to as *Carnap's (categoricity) problem* in reference to Carnap's discovery in [6] that the classical single-conclusion rules of the usual constants of FOL underdetermine their standard model-theoretic interpretations. This problem draws attention to the fact that the inferential behaviour of most constants in most logical systems is consistent with a variety of model-theoretic interpretations. As a result, the model-theoretic interpretations of these notions remain underdetermined by their inferential roles. While *Carnap's Problem* has received renewed interest in the last couple of years ([10], [9], [4], [14]) investigations into its scope and extent for expressions from the category of *generalized quantifiers* are still in its infancy. In this talk, I want to outline a general framework for investigating the question of determinacy for generalized quantifiers and present and discuss some initial results.

In a recent paper, Bonnay & Westerståhl [3] showed that the standard consequence relation of FOL indeed determines the standard interpretations of the universal and existential quantifier as long as the demand that quantifiers be permutation-invariant is met. In the class of type $\langle 1 \rangle$ quantifiers the ability to be uniquely determined by a first-order consequence relation extends beyond the notions of FOL. In fact, the quantifiers

(i) $\mathcal{Q}_0(M) = \{A \subseteq M \mid \aleph_0 \leq |A|\}$

(ii) $\mathcal{Q}_{fin}(M) = \{A \subseteq M \mid |A| < \aleph_0\}$

are uniquely determinable by their respective consequence relations. That they are is the consequence of a more general result according to which all EC_Δ-definable notions are uniquely determinable by consequence relations in a canonical way, and the fact that unique determination is preserved under taking complements (a fact first observed by D. Westerståhl). On the other hand, despite its logic allowing a complete and recursive axiomatization [8], the quantifier

(iii) $\mathcal{Q}_1(M) = \{A \subseteq M \mid \aleph_1 \leq |A|\}$

is not uniquely determinable by any consequence relation over its language. This indeterminacy continues to persist into cardinalities higher than $\aleph_1$.

If there is time, I would like to discuss (some of) the philosophical consequences of these results further. For, on the one hand, the difference between the unique determinability of $\mathcal{Q}_0$ in the context of a first-order language as against the failure of unique determinability of $\mathcal{Q}_1$, despite the latter's complete logic, throws doubt on the idea that completeness of a logical system is a necessary condition for a 'full understanding' or 'complete grasp' of its notions. This, on the other hand, has interesting implications for a debate in the philosophy of mathematics: it is well-known that the natural number structure can be categorically characterized in the language of FOL extended by the quantifier 'there are (in)finitely many'. The above then provides a way to defend the use of this formalism for achieving determinate reference to the natural number structure against the 'model-theoretic skeptic' in the philosophy of mathematics (see, e.g., [5]) who doubts our ability to achieve, in a naturalistically acceptable fashion, such reference.

This talk is based on joint work with D. Bonnay and discusses material previously published ([2]) and submitted for publication.

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Free p-algebras

Katarzyna Słomczyńska[1], ***Tomasz Kowalski***[2]

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14:30–15:00

We give a new construction of free distributive p-algebras. Our construction relies on a detailed description of completely meet-irreducible congruences, so it is purely universal algebraic. It yields a normal form theorem for p-algebra terms, simpler proofs of several existing results, as well as a complete characterisation of structurally complete varieties of p-algebras.

Sequent Calculi for Two non-Fregean Theories

Agata Tomczyk

14:30–15:00

Adam Mickiewicz University, Faculty of Psychology and Cognitive Science, Department of Logic and Cognitive Science, Poznań, Poland

The aim of the talk is to present Sequent Calculi for two non-Fregean theories: WT—a topological Boolean extension of SCI (*Sentential Calculus with Identity*), the weakest non-Fregean logic proposed by Roman Suszko [2], and WH—an axiomatic extension of WT consisting of formulas tautological in Henle algebras. Non-Fregean theories were introduced as a formalization of ontology of Wittgenstein's *Tractatus* [4], which was intertwined with the abolition of the (so called) Fregean Axiom [2]. Fregean Axiom can be formulated as follows:

$$(\phi \leftrightarrow \chi) \to (\phi \equiv \chi)$$

which states that identity can be identified with classical equivalence, or that semantic correlates of sentences are synonymous with their truth values. Suszko disagreed with this assumption and, instead, following Wittgenstein's ontology, introduced the concept of a *situation* as the denotation of a sentence. This particular idea has been formalized through the introduction of the binary identity connective "≡", which is stronger than classical equivalence "↔" and which expresses that situations denoted by two analyzed sentences are identical. The two non-Fregean logics examined in the talk are two of three main extensions of SCI (the other one being WB) examined by Suszko in *Abolition of the Fregean Axiom* [2]. Moreover, proof theory for non-Fregean logics has been a subject of examination before, but it was mostly focused on SCI and its intuitionistic version, ISCI; the examination of axiomatic extensions of SCI in structural proof theory realm provides a new insight into the topic of formalizing non-Fregean proof systems.

WT is obtained from WB, a Boolean extension of SCI, through addition of four axioms extending the properties of identity connective. As a result the identity is weaker and the overall logic is stronger, but the resulting identity connective is still separate from classical equivalence. WT's philosophical foundations lay in the following proposition from *Tractatus*:

> 5.141 If p follows from q and q from p then they are one and the same proposition.

which can be interpreted as the fact that two logically equivalent sentences constitute different variants of the same sentence. WT is closed under both the Gödel rule and the *quasi-Fregean* rule, that is

$$\frac{\phi \leftrightarrow \chi}{\phi \equiv \chi}$$

WT consists of more tautological identities than SCI and WB, where in the case of SCI the only tautological equation is the trivial one of the form $\phi \equiv \phi$ and within WB the tautological equations $\phi \equiv \chi$ must come from classically provable equivalences $\chi \leftrightarrow \chi$. In the case of WT, $\phi \equiv \chi$ is a tautology if and only if $\phi \leftrightarrow \chi$ is a tautology of SCI. To formalize this notion we introduce proof system $\mathsf{G3}_{\mathsf{WT}}$ (based on a version of the original system $\ell\mathsf{G3}_{\mathsf{SCI}}$ found in [1]), in which we add one right-sided identity-dedicated rule

$$\frac{\Gamma^{\equiv} \Rightarrow \phi \leftarrow \chi}{\Gamma^{\equiv} \Rightarrow \phi \equiv \chi}\ R^{T}_{\equiv}$$

in which $\Gamma^{\equiv}$ consists of equations only. This particular restriction is motivated by the need to prevent the possibility of proving the so-called Fregean Axiom. The resulting system is complete and sound. We will discuss correctness and invertibility of the proposed rule set and identify issues regarding the cut elimination procedure.

WH, an axiomatic extension of WT, differs from WT in the addition of one supplementary identity-dedicated axiom (although it is possible to use the axiomatization of WB as the base—if it was the case, WH would be obtained through the addition of four axioms characterizing identity). It is a formalization of the following proposition from *Tractatus*:

5.5303 Roughly speaking: to say of **two** things that they are identical is nonsense, and to say of **one** thing that it is identical with itself is to say nothing.

which is often quoted to emphasize Wittgenstein's aversion towards the sign of identity. In WH we can state that a given situation is either necessary or impossible. This brings us closer to modal logic; WH corresponds to S5, where $\Box$ can be interpreted as interior operator "I". We will briefly comment on this particular correspondence and then we will introduce $\mathsf{G3}_{\mathsf{WH}}$, sequent calculus obtained from $\mathsf{G3}_{\mathsf{WT}}$ through the extending the set of identity-dedicated rules. The newly added rule is introduced through the utilization of Negri's strategy of turning axioms into sequent calculus rules. As a result we obtain the following rule:

$$\frac{\Gamma, (\phi \equiv \chi) \equiv \top \Rightarrow \Delta \quad \Gamma, (\phi \equiv \chi) \equiv \bot \Rightarrow \Delta}{\Gamma \Rightarrow \Delta}\ L^5_{\equiv}$$

In the above rule we have two active formulae and no principal formula. Its general form is similar to *cut*; when looking bottom-up, we introduce two possible scenarios in our derivation—a given equation $\phi \equiv \chi$ can be either necessary or impossible. The obtained system is complete and sound. Similarly as it was mentioned for $\mathsf{G3}_{\mathsf{WT}}$, we will discuss the correctness and invertibility of the obtained identity-dedicated rule as well as discuss the issues connected to the cut elimination procedure.

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Infinitary semilinear extensions of Abelian logic

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15:30–16:00

It is a well known fact, that the class of Abelian ℓ-groups is a variety. It was established in [2] that even as a quasivariety it is generated by the ℓ-group of integers. In fact, it is known that, as a quasivariety, it is generated by any of its nontrivial members, and therefore has no proper subvarieties, or in the language of logic: Abelian logic has no consistent finitary extensions. It does, however, have consistent *infinitary* extensions, i.e. extensions that can only be axiomatized by rules using ω-many variables. These extensions are in one-to-one correspondence with generalized quasivarieties of Abelian ℓ-groups.

Let us recall the definition of generalized quasivariety and generalized quasiequation.

Definition. *A generalized $\mathcal{L}$-quasiequation is an expression of the form*

$$\bigwedge_{i<\kappa} \varphi_i \approx \psi_i \to \varphi_\kappa \approx \psi_\kappa,$$

where $\varphi_i \approx \psi_i$ are $\mathcal{L}$-equations for every $i < \kappa$, where κ is a countable cardinal number.

A class $\mathbb{K}$ of $\mathcal{L}$-algebras is a generalized quasivariety if there exist a set of generalized quasiequations S such that $\mathbb{K}$ satisfies all generalized quasiequations from S. It is determined by set of generalized quasiequations.

We denote the smallest quasivariety containing class of algebras $\mathbb{K}$ as $\mathbf{GQ}(\mathbb{K})$. For one element algebra $\boldsymbol{A}$ we write $\mathbf{GQ}(\boldsymbol{A})$ instead of $\mathbf{GQ}(\{\boldsymbol{A}\})$.

The first topic of this talk will be study of the generalized quasivariety generated by the ℓ-group of real numbers $\mathbf{GQ}(\boldsymbol{R})$. We will show that $\mathbf{GQ}(\boldsymbol{R})$ is also generated by all Archimedean ℓ-groups. We will axiomatize this generalized quasivariety using the $\vee$-version of Archimedean rule:

$$\{\psi \vee \xi \Rightarrow (n \cdot \varphi) \vee \xi \mid n \in \mathbb{N}\} \blacktriangleright \varphi. \qquad (\mathrm{Arch}_\vee)$$

In the rest of the talk, we will focus on generalized subquasivarieties of $\mathbf{GQ}(\boldsymbol{R})$. We will make first steps towards describing $\mathbf{GQ}(\boldsymbol{Q})$ and $\mathbf{GQ}(\boldsymbol{Z})$ and in particular we will show there are 2^{2^ω} generalized subquasivarieties of $\mathbf{GQ}(\boldsymbol{R})$.

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Mechanical causation as relevant implication

András Kornai[1,2]

15:30–16:00

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We present a system of logic aimed treating causation and deduction by the same means. We take deduction steps to be transitions of finite automata, and we model causation by taking temporal steps, endowing both notions by the same level of (physical) necessity that we attribute to the passing of time.

Thursday 20th

From questions to proofs and back: on questions in proof systems

Dorota Leszczyńska-Jasion

9:00–10:00

Adam Mickiewicz University, Poznań
Department of Logic and Cognitive Science

The talk will provide an overview of the logic of questions in its intersection with proof theory. The general aim is to describe how questions function in proof systems.

In the first part of my talk I will sketch a historical overview of the developments in the logic of questions that are relevant to the topic[1]. I wish to pay special attention to what may be called the 'Polish tradition' in the logic of questions: starting, say, with Kazimierz Ajdukiewicz ([1, 2]) and leading to Inferential Erotetic Logic developed by Andrzej Wiśniewski ([14, 16]). The work of Tadeusz Kubiński ([9, 10, 11])—barely known, for historical reasons, in the Western tradition—is of great importance in this context.

In the second part I shall focus on proof systems including questions that are being developed today. These includes (though are not limited to) natural deduction systems defined in the framework of Inquisitive Semantics ([3, 4]) and the method of Socratic proofs defined in Inferential Erotetic Logic ([15, 16, 12, 13]).

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[1] I shall refer mainly to ([6, 7, 8]), as far as Western literature is concerned.

Epistemic Logics of Structured Intensional Groups: Agents - Groups - Names - Types

Marta Bílková

10:30–11:30

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Work on this paper was supported by the grant no. 22-23022L CELIA of the Czech Science Foundation

In the overwhelming majority of contributions to multi-agent epistemic, doxastic, and coalition logic, a group is reduced to its extension, i.e., the set of its members. Membership in groups is assumed to be common knowledge among all agents. This has a counter-intuitive consequence that groups change identity when their membership changes, and rules out uncertainty regarding who is a member of a given group. Additionally, this idealization does not reflect the structure of groups, or the structured way in which collective epistemic attitudes emerge, in the intended application of logical models. Epistemic logics of intensional groups lift the extensionality assumptions above, by seeing groups as given to us intensionally by a common property that may change its extension from world to world. We will outline an abstract framework replacing agent or group labels of epistemic modalities with names, and providing them with an algebraic structure relevant to types of collective epistemic attitudes in question. The resulting formalisms are essentially two-sorted, combining the language of labels of modalities and the language of epistemic statements. (The talk is grounded in on-going joint work with Zoé Christoff, Olivier Roy, and Igor Sedlár.)

One of the usual assumptions of multi-agent epistemic logic is that groups of agents are given *extensionally* as sets of agents, membership in groups is common knowledge among all agents, and change in membership implies change of identity of a group. This is not how we usually think of groups. We are commonly reasoning in various contexts without knowing groups' extensions—we might routinely refer to groups such as "bot accounts", "democrats", or "correct processes"—and we do not settle for reducing groups to their extensions either, as clearly they can change across the state space of a system, or possible states of the world. Epistemic logics of intensional groups lift the assumptions above, by seeing groups as given to us *intensionally* by a common property that may change its extension from world to world. In their seminal work [5, 4], Grove and Halpern introduced a multi-agent epistemic logic where groups are labeled by abstract *names* whose extensions can vary from world to world. The language contains two types of modalities: $E_n\varphi$ means that "everyone named n knows that φ ", and $S_n\varphi$ means that "someone named n knows that φ ". They further consider a natural extension of the basic framework where names are replaced by formulas expressing *structured* group-defining concepts. Motivated mainly by applications such as dynamic networks of processes, another framework where the agent set can vary from state to state, have been developed in a form of term-modal logic. Introduced by [3], it builds upon first order logic, indexing modalities by terms that can be quantified over. Epistemic logic with names of [5] was in a sense seminal to the development of term-modal logic, and can be seen as its simple decidable fragment (a closely related language of implicitly quantified modal logic was studied in [7]).

Grove and Halpern's work is enjoying a recent resurgence of interest in the epistemic logic community: in [1], we considered expansions with non-rigid versions of common and distributed knowledge; Humml and Schröder [6] generalize Grove and Halpern's approach to structured names represented by formulas defining group membership, including e.g. formulas of the description logic ALC. Their abstract-group epistemic logic (AGEL) contains a common knowledge modality as the only modality and, unlike in [1, 5], their group names are rigid. In [2], we adopted the perspective that both "everyone labeled a knows" and "someone labeled a knows" modalities form a minimal epistemic language for group knowledge where groups are understood intensionally, and that their labels reflect their structured nature. We used languages built on top of classical propositional language containing modalities $[a]$, $\langle a]$ indexed by elements of an algebra of a given signature of interest, we set up a general relational semantics involving an algebra of group labels to index (sets of) relations in each world, shown how some related logics can be modelled in such a way, and proven completeness of the minimal logic.

A fully abstract account of such epistemic logics can be given, linking two-sorted algebras (involving propositions and group labels/types of knowledge) with monotone neighborhood frame semantics, in terms of an algebraic duality. This can further be applied to obtain, e.g., a definability theorem or to design a multi-type proof theory for the basic logic. We further discuss several particular examples of algebraic signatures giving rise to interesting and useful variants of group knowledge, like distributed or common knowledge.

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On a Generalization of all Strong Kleene Generalizations of Classical Logic

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11:30–12:00

Belnap-Dunn semantics include two extra values, "both" and "neither", in addition to the classical "true" and "false". This semantic context allows for a wider characterization of the concepts of *truth* and *falsity*. [4] studies all the logical consequences that can be defined upon these new characterizations providing semantic and tableau analysis for all of them. This paper extends Wintein's work by providing the corresponding semantic and tableau analysis of metainferences for these *Strong Kleene Generalizations of Classical Logic*.

Our semantics includes four truth-values, t, b, n and f, informally standing for 'true', 'both true and false', 'neither true nor false' and 'false', respectively. In this context, each one of the concepts of *truth* and *non-falsity* split into *exact* and *regular*. Exact truth refers to value t, while regular truth means taking a value in $\{t, b\}$, and similarly for falsity. Following Wintein, we will use the tags $\mathrm{t}, 1, \hat{0}, \hat{\mathrm{f}}$ for exact truth, regular truth, regular non-falsity and exact non-falsity, respectively. The four notions of truth and non-falsity lead, this way, to sixteen instantiations of (GS). Wintein classifies these instances into four "exact" ones, four "regular" ones and the remaining ones as "mixed".

Now, let $\mathcal{L}$ be a propositional language including the classical connectives: $\wedge$, $\vee$, $\supset$ and $\neg$. Our set of four truth-values $\textsc{Val}$ forms a lattice as shown in Fig. 0.1.

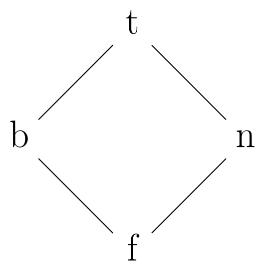

Figure 0.1: Belnap-Dunn four-element lattice

The notions of satisfaction or "standards" (see [1] and [2]) are the upsets of $\textsc{Val}$, and the "antistandards" are the downsets of $\textsc{Val}$, i.e. the complements of an upset of $\textsc{Val}$.

Table 0.1: Standards and antistandards for $\textsc{Val}$.

Standards	Antistandards
$\mathrm{t} = \{t\}$	$\hat{\mathrm{t}} = \{f, n, b\}$
$1 = \{t, b\}$	$\hat{1} = \{f, n\}$
$\hat{0} = \{t, n\}$	$0 = \{f, b\}$
$\hat{\mathrm{f}} = \{t, n, b\}$	$\mathrm{f} = \{f\}$

A valuation v is an **xy-counterexample** to a sequent $A \Rightarrow B$, written $v \nvDash_{xy} A \Rightarrow B$, just in case $v(A) \in x$ and $v(B) \in \hat{y}$ (otherwise, v *satisfies* the sequent). A sequent $A \Rightarrow B$ is **xy-valid**, written $\vDash_{xy} A \Rightarrow B$, when no valuation is an xy-counterexample to it.

Hence, xy-validity involves an upset (x) and a downset ($\hat{y}$) and a sequent $A \Rightarrow B$ is xy-valid exactly when no valuation v sets both $v(A) \in x$ and $v(B) \in \hat{y}$. A salient difference between the upsets and downsets of our sixteen logics is whether they are exclusive and exhaustive with respect to the set of values. We will clarify the relation between these properties and their non-reflexivity and local non-transitivity.

We then develop our tableau system starting from the one used by [3] for FDE, and adapting it to Wintein's SK generalizations by selecting the appropriate tags for formulas in the initial list and a set of rules to transform these tags into signed formulas. Hence, in order to check whether $\vDash_{xy} A_1, \dots A_n \Rightarrow B_1 \dots B_m$ we must write the initial list of a counterexample, and then work out standard x and antistandard $\hat{y}$ into its appropriate combination of signs, as shown in Fig. 0.2.

A, t	$\mathrm{A}, 1$	$\mathrm{A}, \hat{0}$	$\mathrm{A}, \hat{\mathrm{f}}$
$\mathrm{A}, +$ $\neg\mathrm{A}, -$	$\mathrm{A}, +$	$\neg\mathrm{A}, -$	$\mathrm{A}, +$ $\neg\mathrm{A}, -$

A, f	$\mathrm{A}, \hat{1}$	$\mathrm{A}, 0$	$\mathrm{A}, \hat{\mathrm{t}}$
$\neg\mathrm{A}, +$ $\mathrm{A}, -$	$\mathrm{A}, -$	$\neg\mathrm{A}, +$	$\neg\mathrm{A}, +$ $\mathrm{A}, -$

Figure 0.2: Signed formulae for upsets and downsets

We can then extend this method to metainferences, which are expressions of the form $A \Rightarrow B \Rrightarrow C \Rightarrow D$, where $A \Rightarrow B$ and $C \Rightarrow D$ are sequents.

A valuation v is an xy**-counterexample** to a metainference $A \Rightarrow B \Rrightarrow C \Rightarrow D$, written, $v \nVdash_{xy} A \Rightarrow B \Rrightarrow C \Rightarrow D$, just in case $v \Vdash_{xy} A \Rightarrow B$ and $v \nVdash_{xy} C \Rightarrow D$ (v otherwise xy-satisfies the metainference). A metainference $A \Rightarrow B \Rrightarrow C \Rightarrow D$ is xy-valid, $\vDash_{xy} A \Rightarrow B \Rrightarrow C \Rightarrow D$, precisely if no valuation is a counterexample to it.

Our strategy to apply the tree method to metainferences is by extending our initial lists with nodes of the form $A \Rightarrow B, xy$ and $A \Rightarrow B, \overline{xy}$. The tree rules for this kind of nodes are as in Fig. 0.3.

$A \Rightarrow B, xy$	$A \Rightarrow B, \overline{xy}$
$A, \hat{x}$ $\quad B, y$	A, x $B, \hat{y}$

Figure 0.3: Sequent arrow rules

The rules follow the conditions for xy-satisfaction and xy-counterexample for sequents. In order to decide whether the metainference $A \Rightarrow B \Rrightarrow C \Rightarrow D$ is xy-valid we write the initial list for a counterexample, with the premiss sequents tagged with xy and the conclusion sequents with $\overline{xy}$. We then apply the sequent rules and afterwards the translation of standards into signed nodes.

We then show how to adapt soundness and completeness proofs to the new material, proving that our tableaux system for all SK-generalizations is sound and complete with respect to valid metainferences in the four-valued semantics of all SK generalizations.

We conclude with an intriguing dilemma. On the one hand, there seems to be a deep symmetry between the non-reflexive and non-transitive consequence relations, something the *local reading* of metainference validity reflects. That symmetry, however, cannot be accounted for under a *global reading*. On the other hand, some of the logics are "swap" equivalents to each other, meaning that for any logic xy, its **swap** is the result of substituting each 1 by $\hat{0}$ and each $\hat{0}$ by 1 in xy. At the level of valuations, this amounts to the fact that for a given valuation v its **bn-swap** $v^{\dagger}$ is the result of replacing all n's by b's and all b's by n's. Under an intuitive reading of the consequence relations we consider, swap-logics should be equivalent, and in this case, it's the *global reading* of validity the one that provides the desired result, as these "swap" logics come apart under the *local reading*.

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Semiassociative Lambek Calculus: Sequent systems and algebras

Paweł Płaczek

11:30–12:00

WSB Merito University in Poznań, Poland

Lambek Calculus (L), introduced by Lambek [2], is a significant substructural logic. It arises from the omission of three standard structural rules: weakening, contraction, and permutation. The algebraic models of L are termed *residuated semigroups* or *L-algebras*, and are defined as follows.

Definition 2. Let $(M, \otimes, \multimap, \circ\!-, \leq)$ be a structure where $(M, \otimes)$ is a semigroup, $(M, \leq)$ is a poset, and the following condition holds:

$$\text{(RES)}\, a \otimes b \leq c \iff b \leq a \multimap c \iff a \leq c \circ\!- a$$

Then, $(M, \otimes, \multimap, \circ\!-, \leq)$ is a *residuated semigroup.*

Operation $\otimes$ is called *product* and $\multimap, \circ\!-$ are called its *residuations*. As noted, we assume the product is associative. Dropping that assumption leads to the formation of a *residuated groupoid* or an *NL-algebra*.

Residuated groupoids are models of Nonassociative Lambek Calculus (NL) of Lambek [3]. It represents another crucial substructural logic, often described as the pure logic of residuations because the product does not require any properties.

Both of these logics have interesting properties and applications. They were meant to describe categorial grammars. Associative variant would describe sentences as sequences of words (tokens) and nonassociative variant would descibe sentences as syntactic trees.

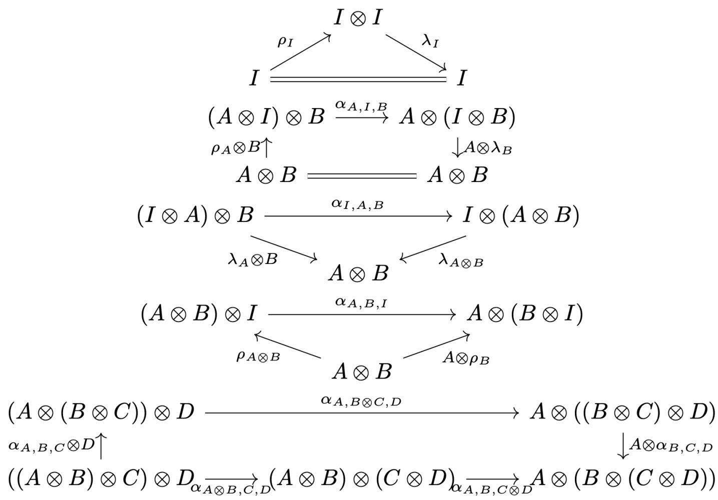
It is common to add a multiplicative constant 1 to L and NL. In algebras, constant 1 is interpreted as a unit for the product. In that way we obtain *residuated monoid* or *residuated unital groupoid*. In this paper we distinct L, NL, L1 and NL1, where the latter two admit the constant. The multiplicative constant requires different approach to sequents.

The crucial difference between L and NL (or L1 and NL1) is their complexity. The problem whether a sequent is provable in L is an NP–complete problem (see Pentus [5]), while the same problem for NL can be solved in polynomial time (see Buszkowski [1]). Moreover, if we consider the consequence relation, i.e. we admit finite set of nonlogical axioms, L becomes undecidable while NL is still PTIME (see Buszkowski [1]). The same remains true for variants with 1.

The undecidability of consequence relation of L is a great problem if we would like, for example, decide whether a sentence in a lanugage is correct. Sentences usually appear as sequences, not trees. Hence, there is a need for a new framework.

Uusatlu et al [7] introduced Skew Noncommutaive Multiplicative Intuitionistic Linear Logic (SkNMILL). They started with the following definition by Street [6].

Definition 3 (cited from [7, 6])**.** A (left) skew monoidal closed category $\mathbb{C}$ is a category with a unit object I and two functors $\otimes : \mathbb{C} \times \mathbb{C} \to \mathbb{C}$ and $\multimap$: $\mathbb{C}^{op} \times \mathbb{C} \to \mathbb{C}$ forming an adjunction $- \otimes B \dashv B \multimap -$ for all B, and three natural transformations λ, ρ, α typed $\lambda_A : I \otimes A \to A$, $\rho_A : A \to A \otimes I$ and $\alpha_{A,B,C} : (A \otimes B) \otimes C \to A \otimes (B \otimes C)$, satisfying the following equations due to Mac Lane [4]:



If we assume natural transformations λ, ρ and α are natural isomorphism, then we obtain *monoidal closed category*; see Mac Lane [4].

Monoidal closed categories are models for L1 with only one residuation and are category theory alternative for (restricted) residuated monoids. Uusatlu et al [7] introduce logic for skew monoidal closed categories (i.e. SkNMILL) and provide a sequent system.

The research questions one can state are the following: What are algebraic alternatives for skew variants? How can we introduce second residuation? What is the relation between NL, L (NL1, L1) and their *skew* version? All these questions will be answered in this paper.

In this paper we introduce Semiassociative Lambek Calculus (SL and SL1) which is an extension of SkNMILL with the second residuation. We provide a sequent system for this logic, we prove completeness with algebraic models and show a way to contruct them.

We not only add second residuation, but also additive connectives $\vee$ and $\wedge$ just like Veltri and Wan [8]. Lambek Calculus with these connectives is often called Full Lambek Calculus (FL, FL1). In this paper we consider FSL and FSL1. The additive connectives do not play a major role in proofs, so all the results remain true for weaker logics – SL and SL1.

What are algebraic alternatives for skew variants?

Definition 4. Let $(M, \otimes, \multimap, \multimapinv, \leq)$ be a structure such that $(M, \otimes)$ is a groupoid and $(M, \leq)$ is a poset and the following conditions hold:

$$\text{(L-ASS)}\ (a \otimes b) \otimes c \leq a \otimes (b \otimes c)$$
$$\text{(RES)}\ a \otimes b \leq c \iff b \leq a \multimap c \iff a \leq c \multimapinv a$$

Then, $(M, \otimes, \multimap, \multimapinv, \leq)$ is a *residuated semiassociative groupoid* or *SL-algebra*.

The structure $(M, \otimes, \multimap, \multimapinv, 1, \leq)$ is a *residuated semiassociative unital groupoid* or *SL1-algebra*, if $(M, \otimes, \multimap, \multimapinv, \leq)$ is an SL-algebra and for all $a \in M$ we have:

$$1 \otimes a = a = a \otimes 1$$

As we can see, the associativity is weakened to be only less or equal. One may consider:

$$\text{(R-ASS)}\ a \otimes (b \otimes c) \leq (a \otimes b) \otimes c$$

The structure obtained that way is definable in terms of SL-algebras (one need to define new $\otimes$ with reversed arguments).

The SL-algebras are models of SL and SL1-algebras are models for SL1. To obtain models for FSL and FSL1, we require $\leq$ to be lattice order and add meet and join as new operations (additive connectives).

How can we introduce second residuation?

Let $\mathcal{V}$ be an arbitrary set of propositional variables. We define the formulas:

$$\mathcal{F} ::= \mathcal{V} \mid 1 \mid \mathcal{F} \otimes \mathcal{F} \mid \mathcal{F}/\mathcal{F} \mid \mathcal{F}\backslash\mathcal{F} \mid \mathcal{F} \wedge \mathcal{F} \mid \mathcal{F} \vee \mathcal{F}$$

A sequent is of the form $\sigma \mid \Gamma \Rightarrow C$, where σ is a formula or is empty (denote: ϵ), Γ is a sequence of formulas and C is a formula. From now on, by Greek capital letters we denote sequences of formulas, by Greek lower-case letters we denote a formula or ϵ, by Latin capital letters we denote formulas (always nonempty); Latin lower-case letters are reserved for propositional variables. The following rules are from Uusatlu et al [7] and Veltri and Wan [8]:

$$\frac{}{A \mid \epsilon \Rightarrow A}\ (\text{ax}) \qquad \frac{A \mid \Gamma \Rightarrow C}{\epsilon \mid A, \Gamma \Rightarrow C}\ (\text{pass})$$

$$\frac{A \mid B, \Gamma \Rightarrow C}{A \otimes B \mid \Gamma \Rightarrow C}\ (\otimes \Rightarrow) \qquad \frac{\sigma \mid \Gamma \Rightarrow A \qquad \epsilon \mid \Delta \Rightarrow B}{\sigma \mid \Gamma, \Delta \Rightarrow A \otimes B}\ (\Rightarrow \otimes)$$

$$\frac{\epsilon \mid \Gamma \Rightarrow B \qquad A \mid \Delta \Rightarrow C}{A/B \mid \Gamma, \Delta \Rightarrow C}\ (/ \Rightarrow) \qquad \frac{\sigma \mid \Gamma, B \Rightarrow A}{\sigma \mid \Gamma \Rightarrow A/B}\ (\Rightarrow /)$$

$$\frac{\sigma \mid \Gamma \Rightarrow A \qquad A \mid \Delta \Rightarrow C}{\sigma \mid \Gamma, \Delta \Rightarrow C}\ (\text{s-cut}) \qquad \frac{\epsilon \mid \Gamma \Rightarrow A \qquad \sigma \mid \Delta_1, A, \Delta_2 \Rightarrow C}{\sigma \mid \Delta_1, \Gamma, \Delta_1 \Rightarrow C}\ (\text{c-cut})$$

$$\frac{\epsilon \mid \Gamma \Rightarrow C}{1 \mid \Gamma \Rightarrow C}\ (1 \Rightarrow) \qquad \frac{}{\epsilon \mid \epsilon \Rightarrow 1}\ (\Rightarrow 1)$$

$$\frac{A \mid \Gamma \Rightarrow C \qquad B \mid \Gamma \Rightarrow C}{A \vee B \mid \Gamma \Rightarrow C}\ (\vee \Rightarrow) \qquad \frac{\sigma \mid \Gamma \Rightarrow A}{\sigma \mid \Gamma \Rightarrow A \vee B} \quad \frac{\sigma \mid \Gamma \Rightarrow B}{\sigma \mid \Gamma \Rightarrow A \vee B}\ (\Rightarrow \vee)$$

$$\frac{A \mid \Gamma \Rightarrow C}{A \wedge B \mid \Gamma \Rightarrow C} \quad \frac{B \mid \Gamma \Rightarrow C}{A \wedge B \mid \Gamma \Rightarrow C}\ (\wedge \Rightarrow) \qquad \frac{\sigma \mid \Gamma \Rightarrow A \qquad \sigma \mid \Gamma \Rightarrow B}{\sigma \mid \Gamma \Rightarrow A \wedge B}\ (\Rightarrow \wedge)$$

And the following rule allows to introduce second residuation.

$$\frac{B \mid \Delta \Rightarrow C \qquad \sigma \mid \Gamma \Rightarrow A}{\sigma \mid \Gamma, A\backslash B, \Delta \Rightarrow C}\ (\backslash \Rightarrow) \qquad \frac{A \mid \Gamma \Rightarrow B}{\epsilon \mid \Gamma \Rightarrow A\backslash B}\ (\Rightarrow \backslash)$$

These rules collectively define the sequent system for FSL1. If we skip additive connectives and rules for them, we obtain a sequent system for SL1.

A sequent system for FSL and SL requires a different approach. We do not admit the unit 1 in the language and its rules. Also, we assume that $\sigma \mid \Gamma \Rightarrow C$ is a sequent, if at least one of σ, Γ is nonempty.

Definition 5. A *valuation* μ is a homomorphism from the free algebra of formulas into an FSL1-algebra. The homomorphism is extended to the sequences of formulas inductively as follows:

$$\mu(A_1, A_2, \ldots, A_n) = \begin{cases} 1, \text{ if } n = 0, \\ \mu(A_1), \text{ if } n = 1, \\ \mu(A_1, \ldots, A_{n-1}) \otimes \mu(A_n), \text{ otherwise} \end{cases}$$

We say that the sequent $\sigma \mid \Gamma \Rightarrow C$ is valid if $\mu(\sigma, \Gamma) \leq \mu(C)$.

The definition of valuation for FSL–formulas assumes that the antecedent is nonempty, so 1 is redundant.

Theorem 6. *FSL is sound and complete with respect to FSL-algebras and FSL1 is sound and complete with FSL1-algebras.*

What is the relation between NL, L and SL?

It is clear that (F)NL is essentially weaker than (F)SL. Every (F)L-algebra is also (F)SL-algebra and it is obvious that there are (F)NL-algebras which are not (F)L-algebras. Less obvious is the fact, that there exist (F)SL-algebras which are not (F)L-algebras. Having weaker assumptions does not necessarly mean, that the associativy cannot be derived from other assumptions. We show the way of constructing (F)SL-algebras from ordered semiassociative unital groupoids and we construct a concrete example, where semiassociativity holds while associativity not.

The same remains true for variants with 1 and the construction of algebras with a unit also is shown.
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Multi-relation Agassiz sums of algebras

Francesco Paoli

14:00–15:00

Department of Pedagogy, Psychology, Philosophy, Universita di Cagliari

Płonka sums are a powerful technique for the representation of algebras in regular varieties. However, certain representations of algebras in *ir*regular varieties — like Polin's variety or the variety of pseudocomplemented semilattices — bear striking similarities to Płonka sums, although they differ from them in some important respects.

We aim at finding a convenient umbrella under which all these constructions, as well as other ones of a similar kind, can be subsumed. We introduce a multi-relation variant of Grätzer and Sichler's Agassiz sums that encompasses Płonka sums as a specialcase. We prove that the above-mentioned representations of Polin algebras and pseudocomplemented semilattices can be recast in terms of sums over appropriate bi-relation Agassiz systems. Finally, we investigate the problem as to which identities are preserved by the construction.

Uniform Weak Kleene Logics

Agustina Borzi[1], *Martina Zirattu*[2]

15:30–16:00

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When working on a multiple-premise and multiple-conclusion setting, i.e. considering inferences between a set of premises and a set of conclusions, in most logical systems the comma on the premise side matches the behavior of the object language conjunction, whilst on the conclusion side it matches that of the disjunction. Namely, an inference of the form $\Gamma \Rightarrow \Delta$ can simply be read as $\bigwedge \Gamma \Rightarrow \bigvee \Delta$, where $\bigwedge \Gamma$ is the conjunction of all formulas in Γ and $\bigvee \Delta$ is the disjunction of all formulas in Δ.

However, this is not the case for Paraconsistent Weak Kleene logic (**PWK**) nor for its paracomplete dual, known simply as Weak Kleene logic $\mathbf{K_3^w}$ ([4]).

Both logical systems, which we will call *WK logics* for short, have a three-valued semantics in which the non-classical truth-value behaves in an "infectious" way, *i.e.* as an absorbing or zero element for all the operations ([8]).

It's easy to check that Simplification ($\phi \wedge \psi \vDash \phi\ (\psi)$) is not valid in **PWK**, whereas $\phi, \psi \vDash_{\mathbf{PWK}} \phi\ (\psi)$. And dually, Addition ($\phi\ (\psi) \vDash \phi \vee \psi$) isn't valid in $\mathbf{K_3^w}$, while $\phi\ (\psi) \vDash_{\mathbf{K_3^w}} \phi, \psi$. This fact shows that the metalinguistic comma alluded earlier doesn't behave as the expected binary associative connective featured in the language ([2]). This suggests that there are two distinct ways of conjoining (disjoining) formulae in WK logics: one can either use conjunction (disjunction) or the comma to gather premises (conclusions) together in **PWK** ($\mathbf{K_3^w}$), resulting in a different set of validities in each case.

The peculiar behaviour of the WK connectives has been source of discussion in the literature, at least for two reasons. First, operators as $\wedge_{\mathbf{WK}}$ and $\vee_{\mathbf{WK}}$ invalidate inference rules usually regarded as constitutive of the meaning of conjunction and disjunction (simplification and addition, respectively). Second, their peculiar truth-conditions have lead many to call into question whether $\wedge_{\mathbf{WK}}$ and $\vee_{\mathbf{WK}}$ can be called a conjunction and a disjunction. For example, [3] calls the first a '*disjunction in disguise*', whilst [9] conclude that the second operator '*is not disjunction*'.

We believe there is another reason one could have for questioning the status of $\wedge_{\mathbf{WK}}$ and $\vee_{\mathbf{WK}}$ as logical connectives.

According to [7], logical constants serve to make explicit in the object language certain structural features of the relation of logical consequence. In the author's words:

> When logical constants are introduced they serve, so to speak, as **punctuation marks** of the object language, for some structural features of deductions. (p. 366)

Thus, operational rules are translation rules from the structural level to the object language *via* logical operators. In this view, a constant can be considered *logical* only if it can be ultimately analyzed in structural terms.

However, it seems that the conjunction of **PWK** and the disjunction of $\mathbf{K_3^w}$ cannot be analyzed on a structural level. The reason for this is that, as it has already been pointed out, the comma cannot represent the behaviour of these operators.

Under this reading we are then left in the uncomfortable position according to which these connectives aren't logical constants.

Therefore, our objective here is to find a solution to this problem, exploring a way of restoring the expected correspondence between metalanguage and object language, namely between commas and conjunction/disjunction in WK logics.

With this purpose we define two logical systems, which we will call **uPWK** and $\mathbf{uK_3^w}$ (*uWK logics*, which stands for '*uniform* WK logics'), that are meant to achieve the harmony between meta and object language, and turn out to be substructural: Left Weakening fails in **uPWK**, as does Right Weakening in $\mathbf{uK_3^w}$.

To study these logics, we first redefine the logical consequence relation of WK logics, in order to change the reading of the comma and to bunch together formulas in accordance with the WK truth-tables.

Definition 7 (**uPWK**-validity)**.** $\Gamma \vDash_{\mathbf{uPWK}} \Delta \iff \bigwedge \Gamma \vDash_{\mathbf{PWK}} \bigvee \Delta$

Definition 8 ($\mathbf{uK_3^w}$-validity)**.** $\Gamma \vDash_{\mathbf{uK_3^w}} \Delta \iff \bigwedge \Gamma \vDash_{\mathbf{WK}} \bigvee \Delta$

Then we characterize the logical consequence of both systems with respect to **PWK** and $\mathbf{K_3^w}$, in the following way:

Theorem 9 (Characterization of $\vDash_{\mathbf{uPWK}}$)**.**

$$\Gamma \vDash_{\mathbf{uPWK}} \Delta \; \textit{iff} \begin{cases} \textit{(i)}\ \exists \Delta' \subseteq \Delta,\ s.\ t.\ \varnothing \vDash_{\mathbf{PWK}} \Delta' \quad \textit{or} \\ \textit{(ii)}\ \Gamma \vDash_{\mathbf{PWK}} \Delta \ \textit{and}\ Var(\Gamma) \subseteq Var(\Delta) \end{cases} \tag{0.1}$$

Theorem 10 (Characterization of $\mathbf{uK_3^w}$)**.**

$$\Gamma \vDash_{\mathbf{uK_3^w}} \Delta \; \textit{iff} \begin{cases} \textit{(i)}\ \exists \Gamma' \subseteq \Gamma,\ s.\ t.\ \Gamma' \vDash_{\mathbf{K_3^w}} \varnothing \quad \textit{or} \\ \textit{(ii)}\ \Gamma \vDash_{\mathbf{K_3^w}} \Delta \ \textit{and}\ Var(\Delta) \subseteq Var(\Gamma) \end{cases} \tag{0.2}$$

Additionally, we obtain sequent calculi for uWK logics. To achieve this, we first provide two new sequent calculi for **PWK** and $\mathbf{K_3^w}$, named $\mathcal{S}_{\mathbf{PWK}}$ and $\mathcal{S}_{\mathbf{K_3^w}}$. Contrary to the sequent calculi by [5], $\mathcal{S}_{\mathbf{PWK}}$ and $\mathcal{S}_{\mathbf{K_3^w}}$ do not make use of linguistic restrictions. Instead, they make use of impure rules, i.e. rules that govern more than one connective. Moreover, these proof systems are two-sided, hence, contrary to the existing three-sided sequent calculi by [2], not only look more familiar, but most importantly, do not rely on any particular semantics. $\mathcal{S}_{\mathbf{PWK}}$ and $\mathcal{S}_{\mathbf{K_3^w}}$ (which can also be found in [6]) can be obtained by those in [2] following the method by [1]. Their method guarantees that from sound and complete n-sided calculi one can obtain equivalent two-sided calculi. This allows us to infer the soundness and completeness of $\mathcal{S}_{\mathbf{PWK}}$ and $\mathcal{S}_{\mathbf{K_3^w}}$ from those of the calculi by [2].

Then we derive the sequent calculi for **uPWK** and $\mathbf{uK_3^w}$ from $\mathcal{S}_{\mathbf{PWK}}$ and $\mathcal{S}_{\mathbf{K_3^w}}$, removing the unsound Weakening rules, Left and Right Weakening respectively, and substituting both of them with two new rules that restore a restricted form of weakening requiring some form of variable inclusion (left variable inclusion in the first case, right variable inclusion in the second case).

In conclusion, the new logical systems investigated in this work can be seen as a way to take seriously the problematic status of WK connectives and address it from a substructural point of view. Moreover, the results presented open some new interesting questions. In fact, the same motivation that leads us to the study of uWK logics as solutions to the puzzling mismatch between metalanguage and object language in WK logics, points to a generalization of this strategy for other logics that share the same problem. In particular, we believe that it is promising to extend the study of $uniform$-companions to the so called *Pure Variable Inclusion companions* of Classical Logic, studied in [10].

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On the normal form of deductions in sequent calculus for intuitionistic logic

Ren-June Wang

15:30–16:00

Department of Philosophy, National Chung-Cheng University

Gentzen's *Hauptsatz*, the cut elimination theorem, states that every classical and intuitionistic sequent deduction can be turned into a deduction without a cut. This result and the normalization theorem for natural deduction systems are the cornerstones of structural proof theory. But, in a sense, it is only a limited result. Unlike the normalization theorem, which can be applied to proofs with assumptions, cut elimination only can be applied to sequent deductions in which no assumption, or hypothetical sequent, is present. In this paper, we will take the challenge to generalize the cut elimination theorem for intuitionistic sequent calculus. Our overall goal is to show that every sequent deduction with or without assumptions can be turned into a normal form in which cuts are organized in a certain way such that, among other things, every eliminable cut is eliminated. As such, when our normalization procedure applies to deductions without assumptions, it is a cut elimination procedure. This procedure will then differ from other cut elimination procedure in many ways. It is a kind of bottom-up procedure, which first turns the lower part of a deduction with cuts into its normal form. Furthermore, it is just like the normalization procedure for natural deduction systems, composed of a series of simple reduction steps for the rearrangement of cuts. For example, the following is one of these reductions:

$$\dfrac{\Gamma_1 \Rightarrow \mathrm{A} \qquad \dfrac{\mathrm{A}, \mathrm{A}, \Gamma_2 \Rightarrow \Delta_2}{\mathrm{A}, \Gamma_2 \Rightarrow \Delta_2}}{\Gamma_1, \Gamma_2 \Rightarrow \Delta_2}$$

$$\rightsquigarrow \quad \dfrac{\dfrac{\Gamma_1 \Rightarrow \mathrm{A} \qquad \dfrac{\Gamma_1 \Rightarrow \mathrm{A} \qquad \mathrm{A}, \mathrm{A}, \Gamma_2 \Rightarrow \Delta_2}{\mathrm{A}, \Gamma_1, \Gamma_2 \Rightarrow \Delta_2}}{\Gamma_1, \Gamma_1, \Gamma_2 \Rightarrow \Delta_2}}{\Gamma_1, \Gamma_2 \Rightarrow \Delta_2}$$

The deductions where a cut formula is the principal formula of a contraction—generally the most difficult case that a cut elimination procedure needs to handle—will be directly addressed by this reduction in our process.

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Friday 21st

When change describes time.
Five logics of change for temporal reductionists

9:00–10:00

Kordula Świętorzecka

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The dispute between A- and B-theorists has continued ever since McTaggart formulated his argument for the unreality of time, in 1908. The paradigm of contemporary temporal logic allows Prior-like logics and their modal relational semantics to be treated in a selected sense as A- and B- approaches to time. Interestingly, the A/B opposition divides philosophers of time differently than the opposition between temporal reductionism and absolutism. By temporal reductionism we mean the view that time is dependent on change, or more precisely, that there is no passage of time unless there is any change. What we want to do here is to give a formal frame for a few versions of this position. We will present five different modal logics with different primitive A-operators describing dichotomic changes of different types: $\mathcal{C}$ read *it changes whether*, $\exists\mathcal{C}$ - *it may change whether*; $\forall\mathcal{C}$ - *it must change whether*, $\widetilde{\mathcal{C}}$ - *otherwise it would change whether*. We will interpret the languages of our logics in B semantics with different types of succession relation respectively: linear, cyclic, parallel, and branched. We will give complete axiomatizations for the logics we consider. Finally, we will show an example of the application of one of the formulated logics. It will be extended to the two-sorted theory of change, which is intended to describe a reductionist position according to which the occurrence of changes induces a discrete, linear, and directed flow of time.

A Logic of Probability Dynamics

Ondrej Majer, Igor Sedlár

10:30–11:00

Institute of Computer Science, Czech Academy of Sciences
The work of both authors was supported by the grant GA22-16111S of the Czech Science Foundation.

Building on the work of Hájek et al. [1, 2, 4], we introduce a version of (crisp) modal Łukasiewicz logic [3] that is suitable for formalising reasoning about *probability dynamics*, i.e. processes that lead to a change in the subjective probabilities that agents assign to events. In our model, such processes are modelled as state transitions endowed with a semiring structure.

Probability and its dynamics

One of the central questions in applications of probability theory is the question of learning – how a rational agent should update their subjective probability in the light of new information. There is a large amount of literature devoted to the logical modeling of different types of probability updates, with much of the work focusing on specific models of learning (e.g. Bayesian updating).

We present a more general model of probability updating based on the assumption that probability measures change as a result of state transitions (or actions). The advantage of this approach is that actions (state transitions) need not be (fully) specified in terms of the information received by an agent. In many cases, such a specification is either impossible or difficult to obtain. Furthermore, our approach allows to represent updates resulting from different types of learning methods or situations where the particular method is not known. At the same time, our framework is flexible enough to capture particular methods of learning, e.g. the situations in which a transition consists in receiving a particular piece of information (e.g. when a statement is truthfully announced and the agent learns it with certainty).

Definition 11. Let X be a Boolean algebra and I a set of agents. A *subjective probability measure* on (I, X) is a function $\mu : I \times X \to [0, 1]$ such that

$$\mu_i(1_X) = 1 \qquad (M1)$$
$$\mu_i(x \vee y) = \mu_i(x) + \mu_i(y) \quad \text{if } x \wedge y = 0_X \qquad (M2)$$

Let $\mathbf{M}_I(X)$ be the set of all subjective probability measures on (I, X).

Informally, X is a Boolean algebra of *events* and I a set of agents. The real number $\mu_i(x)$ expresses agent i's subjective assessment of likelihood (*subjective probability*) of the event x. Below we will refer to subjective probability measures simply as probability measures.

Definition 12. Let K be a set of "atomic actions". The set Act_K of *action expressions* based on K is defined using the following grammar:

$$\alpha, \beta ::= a \in K \mid \alpha;\beta \mid \alpha \cup \beta \mid 1_K \mid 0_K$$

The action $\alpha;\beta$ represents *sequential composition* of actions α and β ("do α and then do β"), action $\alpha \cup \beta$ represents *non-deterministic choice* between α and β ("do α or β"), 1_K represents the trivial action ("do nothing" or "wait") and 0_K represents "abort". A natural interpretation of action expressions is in terms of *state transitions*, or binary relations on a set of states:

Definition 13. A *K-frame* is a pair $\langle S, R\rangle$ where S is a non-empty set ("states") and R is a function from Act_K to binary relations on S such that

- $R(\alpha;\beta) = R(\alpha) \circ R(\beta)$;
- $R(\alpha \cup \beta) = R(\alpha) \cup R(\beta)$;
- $R(1_K)$ is the identity relation on S;
- $R(0_K)$ is the empty set.

Note that actions may be non-deterministic: one may have $R(\alpha)st$ and $R(\alpha)su$ without $t = u$. In what follows, we will usually write Act instead of Act_K and R_α instead of $R(\alpha)$.

Definition 14. A *dynamic probability measure* on $\langle S, R\rangle$ is a probability measure on $(S \times I, X)$.

In a dynamic probability measure, a set of states S is given and a probability measure on (I, X) is specified *for each state* in S. States $s \in S$ represent the possible states of the environment and R_α represents the state transitions associated with action α. The central idea captured by this framework is that probabilities can change as a result of action: we may have $R_\alpha st$ and $\mu_{s,i}(x) \neq \mu_{t,i}(x)$.

Łukasiewicz logic

Łukasiewicz logic is one of the three basic fuzzy logics designed to formalize reasoning with vague notions; see [4].

Definition 15. Let N be a countable set of indices. The *Łukasiewicz propositional language* over N is $\mathcal{L}_N$, defined by the following grammar:

$$\phi, \psi ::= P_j \mid 0 \mid \neg\phi \mid \phi \oplus \psi$$

for $j \in N$. We define $1 := \neg 0$, $\phi \to \psi := \neg\phi \oplus \psi$, $\phi \ominus \psi := \neg(\phi \to \psi)$.

Definition 16. A *Łukasiewicz model* for $\mathcal{L}_N$ is a function $M : \mathcal{L}_N \to [0,1]$ such that $M(0) = 0, M(\neg\phi) = 1 - M(\phi), M(\phi \oplus \psi) = \min(M(\phi) + M(\psi), 1)$.

The set of all Łukasiewicz models for N is denoted as $\boldsymbol{Ł}_N$. A formula $\phi \in \mathcal{L}_N$ is a consequence of (or follows from) a finite set $\Gamma \subseteq \mathcal{L}_N$ over $\boldsymbol{Ł}_N$ iff, for all $M \in \boldsymbol{Ł}_N$, if $M(\psi) = 1$ for all $\psi \in \Gamma$.

Recall that Łukasiewicz implication $\phi \to \psi$ may be seen as expressing a (vague) statement "The truth degree of ϕ is *not much higher* that the truth degree of ψ". Consequently, if $M(\phi) > M(\psi)$, then $M(\phi \ominus \psi)$ is the difference between $M(\phi)$ and $M(\psi)$. This will be crucial for expressing probability comparisons.

Our logic of probability dynamics builds on a modal extension of propositional Łukasiewicz logic based on "crisp" relational frames; see [3, 5, 6]. In our setting, the set of modalities Act_K is countably infinite and structured (with operators ;, $\cup$ and constants 1_K and 0_K), but this difference is not significant in view of the "reduction axioms" discussed below.

Definition 17. Let N be a countable set of indices and let K be a countable set of action letters. The *Łukasiewicz modal language* over N and K is $\mathcal{L}_{N,K}$, defined by the following grammar:

$$\phi, \psi ::= P_j \mid 0 \mid \neg\phi \mid \phi \oplus \psi \mid [\alpha]\phi$$

for $j \in N$ and $\alpha \in Act_K$ (see Section). We define the other Łukasiewicz operators as above. Moreover, we define $\langle\alpha\rangle\phi := \neg[\alpha]\neg\phi$.

Definition 18. A *modal Łukasiewicz model* for N and K is a triple $\mathfrak{M} = \langle S, R, V\rangle$ where $\langle S, R\rangle$ is a K-frame and $V : \mathcal{L}_{N,K} \times S \to [0,1]$ such that, for all $s \in S$,

$$V(0, s) = 0 \qquad V(\phi \oplus \psi, s) = \min(V(\phi, s) + V(\psi, s), 1)$$
$$V(\neg\phi, s) = 1 - V(\phi, s) \qquad V([\alpha]\phi, s) = \min\{V(\phi, t) \mid R_\alpha st\}$$

A *pointed model* is a pair $(\mathfrak{M}, s)$. A formula ϕ is *satisfied in a pointed model* $(\mathfrak{M}, s)$ iff $V(\phi, s) = 1$ in $\mathfrak{M}$ (notation: $(\mathfrak{M}, s) \models \phi$). A formula ϕ is *valid in a model* $\mathfrak{M}$ iff $V(\phi, s) = 1$ in all s in $\mathfrak{M}$ (notation: $\mathfrak{M} \models \phi$. The class of all modal Łukasiewicz models for N, K will be denoted as $\boldsymbol{KŁ}_N$. Let $\boldsymbol{K} \subseteq \boldsymbol{KŁ}$. A formula ϕ is *valid in* $\boldsymbol{K}$ iff it is valid in all models in $\mathbf{K}$. For arbitrary finite $\Gamma \cup \{\phi\} \subseteq \mathcal{L}_{K,N}$, we say that ϕ is a *local consequence* of Γ over $\boldsymbol{K} \subseteq \boldsymbol{KŁ}$ iff $(\mathfrak{M}, s) \models \Gamma$ only if $(\mathfrak{M}, s) \models \phi$, for all $\mathfrak{M} \in \boldsymbol{K}$ and all s in $\mathfrak{M}$ (notation: $\Gamma \models_{\boldsymbol{K}} \phi$).

Probability in Łukasiewicz logic

We show that Łukasiewicz logic can be used to formalize reasoning about probability. First we recall the propositional logic *FP(Ł)* discussed in [4] and then we introduce *FP(KŁ)*, a modal extension of *FP(Ł)*.

In contrast to the presentation of the logic *FP(Ł)* in [4], the formulas of our language will contain agent indices, but this is not a substantial modification. Hájek construes *FP(Ł)* as a "two-layered" logic in which a *modal operator* "it is probable that" is applied to Boolean formulas only (the first layer) and the resulting formulas are then combined using the connectives of *Ł* (the second layer). In our presentation, this modal perspective is replaced by an equivalent one in which the set of propositional variables is indexed by $I \times Tm$, where Tm is a set of Boolean algebra terms over a fixed countable set of variables. We also use a more abstract semantics for the language which is more convenient to work with and which is equivalent to Hájek's semantics.

Definition 19. The *probability language* $\mathcal{PL}$ is $\mathcal{L}_{I\times Tm}$.

Similarly as in [4], formulas $P_{(i,e)}$, usually written as $P_i e$, are read as "e is probable according to i".

Definition 20. A Łukasiewicz model M is an *FP(Ł)-model* iff the following formulas are valid in M:

$$P_i(e) \leftrightarrow P_i(f) \quad \text{if } e \equiv f \tag{F0}$$

$$P_i(\top) \leftrightarrow 1 \tag{F1}$$

$$P_i(e \vee f) \leftrightarrow (P_i(e) \oplus P_i(f)) \quad \text{if } e \wedge f \equiv \bot \tag{F2}$$

The conjunction of these formulas will be denoted as *FP*, the set of all *FP(Ł)*-models as ***FP(Ł)***.

Now we introduce the modal logic of probability dynamics *FP(KŁ)*, which is a combination of *FP(Ł)* and *KŁ*.

Definition 21. The language $\mathcal{PL}_K$ is $\mathcal{L}_{I\times Tm,K}$.

Definition 22. A $K Ł_{I\times Tm}$-model $\mathfrak{M}$ is an *FP(KŁ)*-model iff $\mathfrak{M} \models FP$. The set of all *FP(KŁ)*-models is denoted as ***FP(KŁ)***.

It follows from the definition that in *FP(KŁ)*-models, every V_s is an *FP(Ł)*-model.

Recall that $V_s([\alpha]\phi)$ is the minimum of the truth degrees of formula ϕ is states accessible from s using action α. Dually, $V_s(\langle\alpha\rangle\phi)$ is the maximum. Hence, the truth degree of $[\alpha]P_i e$ in a given pointed model expresses the *minimal possible probability* of e (according to i) after the execution of action α. Similarly, $\langle\alpha\rangle P_i e$ expresses the *maximal possible probability* of e (for i) after α.

Several other examples of formalization will be discussed.

Decidability and completeness

Our main technical results are decidability of local consequence over ***FP(KŁ)*** and a sound and complete axiomatization of the set of formulas valid in ***FP(KŁ)***.

Theorem 23. *For arbitrary finite* $\Gamma \cup \{\phi\} \subseteq \mathcal{PL}_K$*, it is decidable whether* $\Gamma \models_{\boldsymbol{FP(KŁ)}} \phi$.

The theorem is proved by establishing a reduction of local consequence over ***FP(KŁ)*** to local consequence over ***KŁ***, which is known to be decidable [5].

Theorem 24. *A formula* $\phi \in \mathcal{PL}_K$ *is valid in* ***FP(KŁ)*** *iff it is a theorem of the axiom system* FP(KŁ)*, consisting of the following axiom schemata:*

(i) *axioms of Łukasiewicz propositional logic;*

(ii) *distribution axioms:*

$[\alpha](\phi \to \psi) \to ([\alpha]\phi \to [\alpha]\psi)$;

$[\alpha](\phi \oplus \phi) \leftrightarrow [\alpha]\phi \oplus [\alpha]\phi$;

$[\alpha](\phi \oplus \phi^n) \leftrightarrow [\alpha]\phi \oplus ([\alpha]\phi)^n$;

(iii) *reduction axioms:*

$[\alpha \cup \beta]\phi \leftrightarrow [\alpha]\phi \wedge [\beta]\phi$;

$[\alpha;\beta]\phi \leftrightarrow [\alpha][\beta]\phi$;

$[0_K]\phi \leftrightarrow 1$;

$[1_K]\phi \leftrightarrow \phi$.

(iv) *probability axioms:*

$P_i(e) \leftrightarrow P_i(f)$ *if* $e \equiv f$

$P_i(\top) \leftrightarrow 1$

$P_i(e \vee f) \leftrightarrow (P_i(e) \oplus P_i(f))$ *if* $e \wedge f \equiv \bot$

The inference rules are Necessitation ($\phi/[\alpha]\phi$ for all α) and Modus Ponens.

The axiom system *FP(KŁ)* extends the axiom system *KŁ* of [3] with "reduction axioms" and *FP*. (The axioms of *KŁ* use the notation ϕ^n for $\phi \odot \cdots \odot \phi$ where ϕ appears n-times, and $\equiv$ for Boolean equivalence.) Key lemmas in the completeness proof are the reduction of local consequence over ***FP(KŁ)*** to local consequence over ***KŁ*** mentioned above and the completeness proof of [3] using, in addition, our "reduction axioms".

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Maximally Substructural Classical Logic

Camillo Fiore*, *Bruno Da Ré

10:30–11:00

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Roughly speaking, *structural principles* are principles whose formulation does not require any constant of the object language. Classical logic validates a number of well-known structural principles, such as for instance

$$\text{Id}\ \frac{}{A \Rightarrow A}$$

$$\times\text{Cut}\ \frac{\Gamma \Rightarrow \Delta, A \qquad A, \Sigma \Rightarrow \Pi}{\Gamma, \Sigma \Rightarrow \Delta, \Pi} \qquad\qquad +\text{Cut}\ \frac{\Gamma \Rightarrow \Delta, A \qquad A, \Gamma \Rightarrow \Delta}{\Gamma \Rightarrow \Delta}$$

$$\text{C}{\Rightarrow}\ \frac{A, A, \Gamma \Rightarrow \Delta}{A, \Gamma \Rightarrow \Delta} \qquad\qquad {\Rightarrow}\text{C}\ \frac{\Gamma \Rightarrow \Delta, A, A}{\Gamma \Rightarrow \Delta, A}$$

$$\text{W}{\Rightarrow}\ \frac{\Gamma \Rightarrow \Delta}{A, \Gamma \Rightarrow \Delta} \qquad\qquad {\Rightarrow}\text{W}\ \frac{\Gamma \Rightarrow \Delta}{\Gamma \Rightarrow \Delta, A}$$

Here, $A, B, ...$ are formulas of the relevant object language, $\Gamma, \Delta, ...$ are multisets of formulas, and $\Rightarrow$ represents logical consequence. 'C' stands for 'Contraction', 'W' for 'Weakening', 'Id' for 'Identity', '+Cut' for 'Additive Cut' and '×Cut' for 'Multiplicative Cut'.

In the past few years, we have seen the emergence of various logical systems that are coextensive with classical logic, but in a certain sense invalidate some of the above structural principles. In the model-theoretic framework, these systems are obtained by finding a consequence relation that validates all and only the arguments valid in classical logic, but relative to which some of the structural principles are 'locally invalid', that is, fail to preserve satisfaction at every interpretation. Thus, for instance, Cobreros et. al. [6, 2] and Ripley [4, 5] present system **ST**, where the Cut principles are locally invalid. Also, Rosenblatt [6] presents system **NC**, where not only the Cut principles but also the Contraction principles are locally invalid. Curiously, no system of this kind has been proposed where the principles of Weakening are locally invalid. Such a system seems possible in principle, since there are well-known sequent calculi for classical logic where the rules of Weakening are admissible but not derivable (see, e.g. calculus G3 in Indrzejczak [3, p. 114]).

The contribution of this paper consists of four results. First, we fill the abovementioned gap: we present a system whose consequence relation is coextensive with that of classical logic, but in which the Weakening principles and ×Cut are locally invalid. We call this system **nwCL** (for 'No Weakening Classical Logic'), and obtain it by means of a four-valued non-deterministic semantics and a *sui generis* definition of logical consequence. Second, we use a dual procedure to obtain a system whose consequence relation is also coextensive with that of classical logic, but in which the Contraction principles and +Cut are locally invalid; we call it **ncCL** (for 'No Contraction Classical Logic'). Third, we combine the features of the two systems above to obtain a third system whose consequence relation is still coextensive with that of classical logic, but in which the principles of Cut, Weakening and Contraction are *all* locally invalid; we call it **msCL** (for 'Maximally Substructural Classical Logic'). Fourth, and last, we show that all our procedures can be generalized to any Tarskian logic whatsoever. This means, in particular, that for any Tarskian logic **L** it is possible to define a logic **msL** which is coextensive to **L** but locally invalidates all structural principles except for Id. This last result can be seen as a strong generalization of a recent work by Szmuc [7], who shows that for every Tarskian logic it is possible to define a coextensive system that locally invalidates the Cut principles.

To illustrate the machinery we use, we make a quick and dirty presentation of our systems. We start with **nwCL**. Its four-valued non-deterministic semantics is given by the tables

$\neg$	A
1	$\{0,0^\star\}$
$1^\star$	$\{0,0^\star\}$
0	$\{1,1^\star\}$
$0^\star$	$\{1,1^\star\}$

$\wedge$	1	$1^\star$	0	$0^\star$
1	$\{1,1^\star\}$	$\{1,1^\star\}$	$\{0,0^\star\}$	$\{0,0^\star\}$
$1^\star$	$\{1,1^\star\}$	$\{1,1^\star\}$	$\{0,0^\star\}$	$\{0,0^\star\}$
0	$\{0,0^\star\}$	$\{0,0^\star\}$	$\{0,0^\star\}$	$\{0,0^\star\}$
$0^\star$	$\{0,0^\star\}$	$\{0,0^\star\}$	$\{0,0^\star\}$	$\{0,0^\star\}$

$\vee$	1	$1^\star$	0	$0^\star$
1	$\{1,1^\star\}$	$\{1,1^\star\}$	$\{1,1^\star\}$	$\{1,1^\star\}$
$1^\star$	$\{1,1^\star\}$	$\{1,1^\star\}$	$\{1,1^\star\}$	$\{1,1^\star\}$
0	$\{1,1^\star\}$	$\{1,1^\star\}$	$\{0,0^\star\}$	$\{0,0^\star\}$
$0^\star$	$\{1,1^\star\}$	$\{1,1^\star\}$	$\{0,0^\star\}$	$\{0,0^\star\}$

(As the reader can observe, $1^\star$ and $0^\star$ work as extra copies of the classical values 1 and 0. While the latter will ensure that all classical counterexamples are available, the former will give us counterexamples to selected structural rules.) Given a multiset of formulas Γ, we write $A \in \Gamma$ to mean that A appears in Γ at least once. Then, given a valuation v, $v(\Gamma)$ denotes the set $\{v(A) : A \in \Gamma\}$. Also, if x is one of our semantic values, $|\Gamma_v^x|$ is the number of occurrences of formulas in Γ that receive value x at v. Consequence in **nwCL** is defined as follows:

Definition. $\Gamma \models_{\mathbf{nwCL}} \Delta$ if and only if, for every valuation v, it is not the case that both

(a) $v(\Gamma) \subseteq \{1, 1^\star\}$ and $|\Gamma_v^{1^\star}| \neq 1$
(b) $v(\Delta) \subseteq \{0, 0^\star\}$ and $|\Delta_v^{0^\star}| \neq 1$

So, intuitively, value $1^\star$ only contributes to generate a counterexample to an argument when it appears at least twice in the premises. Dually, value $0^\star$ only contributes to generate a counterexample when it appears at least twice in the conclusions. To see how the system locally invalidates the Weakening principles, consider the instances

$$\frac{p \multimap q}{r, p \multimap q} \qquad\qquad \frac{p \multimap q}{p \multimap q, r}$$

The leftmost instance is counterexemplified by any valuation v such that $v(q) = 0$ and $v(p) = v(r) = 1^\star$. The rightmost instance is counterexemplified by any valuation v such that $v(p) = 1$ and $v(q) = v(r) = 0^\star$. To see how the system invalidates $\times$Cut, consider

$$\frac{p \multimap r, s \qquad s, p \multimap r}{p, p \Rightarrow r, r}$$

This instance is counterexemplified, for instance, by any valuation v such that $v(p) = 1^\star$, $v(r) = 0^\star$ and $v(s) \in \{1, 0\}$.

The second system, **ncCL**, is based on exactly the same tables as the previous one. For notational convenience, we just replace the star $\star$ by a circle $\circ$ (so the four semantic values are now 1, 0, 1° and 0°). Consequence is defined as follows:

Definition. $\Gamma \models_{\mathbf{ncCL}} \Delta$ if and only if, for every valuation v, it is not the case that both

(a) $v(\Gamma) \subseteq \{1, 1^\circ\}$ and $|\Gamma_v^{1^\circ}| \leq 1$
(b) $v(\Delta) \subseteq \{0, 0^\circ\}$ and $|\Delta_v^{0^\circ}| \leq 1$

So, 1° only contributes to generate a counterexample to an argument when it appears exactly once in the premises, and similarly for 0° and the conclusions. To see how the system invalidates the Contraction principles, consider the instances

$$\frac{p, p \multimap q}{p \multimap q} \qquad\qquad \frac{p \multimap q, q}{p \multimap q}$$

Both instances are counterexemplified at any valuation v such that $v(p) = 1^\circ$ and $v(q) = 0^\circ$. As for +Cut, consider the instance

$$\frac{p \dashv r, s \qquad s, p \dashv r}{p \Rightarrow r}$$

It is counterexemplified at every valuation v such that $v(p) = v(s) = 1^\circ$ and $v(r) = 0^\circ$.

Lastly, system **msCL** combines the semantics and definitions of consequence of the two previous systems. Letting $\mathbf{1} = \{1, 1^\star, 1^\circ\}$ and $\mathbf{0} = \{0, 0^\star, 0^\circ\}$, the system has the six-valued non-non-deterministic tables

$\neg$	A
1	$\mathbf{0}$
$1^\star$	$\mathbf{0}$
1°	$\mathbf{0}$
0	$\mathbf{1}$
$0^\star$	$\mathbf{1}$
0°	$\mathbf{1}$

$\wedge$	1	$1^\star$	1°	0	$0^\star$	0°
1	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$
$1^\star$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$
1°	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$
0	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$
$0^\star$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$
0°	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$

$\vee$	1	$1^\star$	1°	0	$0^\star$	0°
1	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
$1^\star$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
1°	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
0	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$
$0^\star$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$
0°	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{0}$	$\mathbf{0}$	$\mathbf{0}$

The definition of consequence is the expectable, namely:

Definition. $\Gamma \models_{\mathbf{msCL}} \Delta$ if and only if, for every valuation v, it is not the case that both

(a) $v(\Gamma) \subseteq \mathbf{1}$, and $|\Gamma_v^{1^\star}| \neq 1$, and $|\Gamma_v^{1^\circ}| \leq 1$
(b) $v(\Delta) \subseteq \mathbf{0}$, and $|\Delta_v^{0^\star}| \neq 1$, and $|\Delta_v^{0^\circ}| \leq 1$

It is easy to see that all the counterexamples to structural rules that we had in systems **nwCL** and **ncCL** are still available here. As announced, in the paper we prove that the three systems presented are all coextensive with classical logic. That is, letting $\models_{\mathbf{CL}}$ stand for the consequence relation of classical logic (defined as usual by means of the Boolean bivaluations), we have

Theorem. $\models_{\mathbf{CL}} = \models_{\mathbf{nwCL}} = \models_{\mathbf{ncCL}} = \models_{\mathbf{msCL}}$

In the last part of the paper, we generalize our previous results to any Tarskian logic. In short, we first show how to define, given any logical matrix $\mathcal{M}$ with a designated value 1 and a non-designated value 0, the non-deterministic matrices $\mathcal{M}^\star$, $\mathcal{M}^\circ$ and $\mathcal{M}^{\star\circ}$; these matrices serve to define **nw-**, **nc-** and **ms-**systems, respectively. Then, we rely essentially on Wójcicki's result according to which every Tarskian consequence relation can be characterized by a class $\mathbb{M}$ of logical matrices (see [8]) to claim that for every Tarskian logic **L** there exist an **nw-**, an **nc-** and an **ms-**counterpart coextensive to it.

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Attention to Attention

***Gaia Belardinelli*[1]*, Ondrej Majer*[2]**

11:00–11:30

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The work of the second author was supported by the grant GF22-23022L of the Czech Science Foundation.

Attention is the capacity of the mind to select a portion of the available information and allow that portion only to be discovered and learnt [1]. An agent may focus her attention on portions of the external environment as well as on other agents, and for example learn whether they are paying attention to the same portion of the environment or whether they are paying attention to each other paying attention to that portion. The present work focuses on this capacity for *social attention*, that is, on the capacity to pay attention to aspects of reality together with other agents. Social attention has been extensively studied in the social and cognitive sciences, as some forms of it (i.e., joint attention) appear central to fundamental human capacities such as language acquisition [2]. However, there is still little agreement on the basic mechanisms underlying it and on what it means to pay attention to something *together with somebody else*, with scholars still debating whether and when exactly it gives rise to important group knowledge notions such as common knowledge [3].

In this work, we introduce a formal framework to analyse these social attention mechanisms and their impact on common knowledge by means of epistemic logic. Our framework generalizes a recent dynamic epistemic logic (DEL) model for attention-based learning, where agents can pay attention to a subset of the event happening and learn that subset only [4]. The model in [4] is based on a multi-agent dynamic doxastic language (i.e., a language for propositional logic together with a belief modality for each agent and a dynamic modality) extended with *attention atoms* $\mathsf{h}_a p$, for each agent a, expressing that a is paying attention to whether p holds. When $\mathsf{h}_a p$ is true at a state of the model, and p is revealed by an event, agent a learns the truth value of p, whereas when $\mathsf{h}_a p$ is false at a state of the model, a does not learn the truth value of p. In [4], agents can only pay attention to propositional atomic formulas p, that is, to factual information that is not about agents' attention.

Here, we present a generalization of this framework so that agents can also pay attention to the attention attitudes of other agents. First, we extend the language introduced by [4] with new atomic formulas $\mathsf{h}_a\mathsf{h}_b p$, defined for any two agents a, b and expressing that agent a pays attention to whether b is paying attention to whether p holds. This framework allows for an agent a to be paying attention to whether another agent b is attending to some p, while a is not attending to p herself (i.e., it allows that $\mathsf{h}_a\mathsf{h}_b p$ and $\neg\mathsf{h}_a p$ are both true at a state). In this case, our model captures that a learns that b learns *whether* p obtains, without a learning anything about p herself.

Social Attention Joint attention is "the ability to coordinate attention toward a social partner and an object of mutual interest" [3]. In [3], the authors argue that the joint attention notion is used in several different ways, which suggests a need for clarification or refinement of the notion. They thus introduce a hierarchy of social attention notions, ranging from weaker to stronger versions of attending together. One of the key features distinguishing weak and strong forms of social attention is the type of knowledge involved in or obtained through their attentional states. A strong notion of social attention to an event, for example, presupposes the agents to have common knowledge about their joint attentional state, and only when they do, they also obtain common knowledge about the attended event. However, there are types of social attention where agents may not have common knowledge about their joint attentional state, and may instead be recursively *assuming* to have it – as is the case for a social attention notion called *common attention* [3]. Under common attention, the agents may actually only *believe* that they established common knowledge about the jointness of their attention, where their belief might be faulty. This is thus only a weak notion of social attention, not always involving and not always leading to common knowledge of the jointly attended object. Stronger kinds of joint attention presuppose a closer connection between agents which is called *attention contact* [3]. When two agents establish attention contact, they pay attention to each other paying attention to an event in a direct way, different from recursive inference, which establishes common knowledge of this attending together and thus also achieves common knowledge of the jointly attended event.

The notion of attention contact may be relevant for the discussions about various definitions of common knowledge in logic, and in particular for characterizing common knowledge in a finitary way only requiring attentional contact between two agents. Following the literature, we can distinguish two basic approaches to defining common knowledge: standard recursive or fixpoint definitions, and definitions where common knowledge is described as sharing some situations or the environment in a certain public way, by the individuals [5]. We believe that the notion of attention contact might be considered as a candidate for this second notion of common knowledge, as it constitute a special way of sharing some situations with each other, thus leading to a characterization of common knowledge via strong notions of social attention.

In the rest of the abstract, we give a more detailed idea of the logical frameworks that we will use to explore notions of social attention and related common knowledge. We illustrate the first formal framework mentioned above, where agent may pay attention to other agents paying attention to p, for propositional atoms p. In the talk, we will present both frameworks mentioned in the introduction above, together with illustrative examples, their sound and complete axiomatizations, as well as the connection to social attention notions and common knowledge of the attended object.

Language Throughout, we use Ag to denote a finite set of *agents*, At to denote a finite set of *propositional atoms*. We let $H = \{\mathsf{h}_a p \colon p \in At, a \in Ag\}$ denote a set of *attention atoms* and $HH = \{\mathsf{h}_a \mathsf{h}_b p \colon p \in At, a, b \in Ag\}$ denote the set of *attention to attention atoms*. With $p \in At, a, b \in Ag, q \in At \cup H$ and $\mathcal{E}$ being a multi-pointed event model (see below), define the language $\mathcal{L}$ by:

$$\varphi ::= \top \mid p \mid \mathsf{h}_a q \mid \neg\varphi \mid \varphi \wedge \varphi \mid B_a\varphi \mid [\mathcal{E}]\varphi.$$

The attention atom $\mathsf{h}_a p$ reads "agent a is paying attention to whether p", $\mathsf{h}_a \mathsf{h}_b p$ reads "agent a is paying attention to whether agent b is paying attention to p", $B_a\varphi$ reads "agent a believes φ", and the dynamic modality $[\mathcal{E}]\varphi$ reads "after $\mathcal{E}$ happens, φ is the case".

The formulas in $At \cup H \cup HH \cup \{\top\}$ are called the *atoms*, and a *literal* is an atom or its negation. We often write $\bigwedge S$ to denote the conjunction of a non-empty set of formulas S. To keep things simple, in this work we assume that all consistent conjunction of literals are in a normal form: (i) each atom occurs at most once; (ii) $\top$ doesn't occur as a conjunct; and (iii) the literals occur in a predetermined order (ordered according to some total order on $At \cup H \cup HH$). This implies that for any disjoint sets of atoms P^+ and P^-, there exists a unique conjunction of literals in normal form containing all the atoms of P^+ positively and all the atoms of P^- negatively. For conjuncts that are *not* on this normal form, we assume them to always be replaced by their corresponding normal form. For any conjunction of literals $\varphi = \bigwedge_{1 \leq i \leq n} \ell_i$ and any literal ℓ, we say that φ *contains* ℓ if $\ell = \ell_i$ for some i, and in that case we often write $\ell \in \varphi$. For an arbitrary formula φ, we let $At(\varphi)$ denote the set of propositional atoms appearing in it. We always denote with p a propositional atomic formulas belonging to At.

Kripke model and DEL Semantically, our framework is based on (pointed) Kripke models and standard DEL machinery, namely event models and product update. The event models we use, however, are *multi-pointed* event models. This is because the event models we introduce below (i.e., the event models for attention to attention) encompass all possible attention configuration with respect to the announced formulas for all agents, mapping out, for each attention configuration obtaining in the pointed Kripke model, which is the update.

Definition 25 (Kripke Model)**.** A *Kripke model* is a tuple $\mathcal{M} = (W, R, V)$ where $W \neq \emptyset$ is a finite set of *worlds*, $R : Ag \to \mathcal{P}(W^2)$ assigns an *accessibility relation* R_a to each agent $a \in Ag$, and $V : W \to \mathcal{P}(At \cup H)$ is a *valuation function*. Where w is the *designated world*, we call $(\mathcal{M}, w)$ a *pointed Kripke model.*

Definition 26 (Event Model)**.** An *event model* is a tuple $\mathcal{E} = (E, Q, pre)$ where $E \neq \emptyset$ is a finite set of *events*, $Q : Ag \to \mathcal{P}(E^2)$ assigns an *accessibility relation* Q_a to each agent $a \in Ag$ and $pre : E \to \mathcal{L}$ assigns a *precondition* to each event $e \in E$. Where $E_d \subseteq E$ is a set of *designated events*, $(\mathcal{E}, E_d)$ is a *multi-pointed event model.*

We sometimes denote event models by $\mathcal{E}$ independently of whether we refer to an event model (E, Q, pre) or a multi-pointed event model $((E, Q, pre), E_d)$.

Definition 27 (Product Update)**.** Let $\mathcal{M} = (W, R, V)$ be a Kripke model and $\mathcal{E} = (E, Q, pre)$ be an event model. The *product update* of $\mathcal{M}$ with $\mathcal{E}$ is the Kripke model $\mathcal{M} \otimes \mathcal{E} = (W', R', V')$ where:

$W' = \{(w, e) \in W \times E \colon (\mathcal{M}, w) \vDash pre(e)\}$,

$R'_a = \{((w, e), (v, f)) \in W' \times W' \colon (w, v) \in R_a \text{ and } (e, f) \in Q_a\}$,

$V'((w, e)) = \{p \in At \cup H \cup HH \colon w \in V(p)\}$.

Given a pointed Kripke model $(\mathcal{M}, w)$ and a multi-pointed event model $(\mathcal{E}, E_d)$, we say that $(\mathcal{E}, E_d)$ is *applicable* in $(\mathcal{M}, w)$ iff there exists a unique $e \in E_d$ such that $\mathcal{M}, w \vDash pre(e)$. In that case, we define the *product update* of $(\mathcal{M}, w)$ with $(\mathcal{E}, E_d)$ as the pointed Kripke model $(\mathcal{M}, w) \otimes (\mathcal{E}, E_d) = (\mathcal{M} \otimes \mathcal{E}, (w, e))$ where e is the unique element of E_d satisfying $(\mathcal{M}, w) \vDash pre(e)$.

Definition 28 (Satisfaction)**.** Let $(\mathcal{M}, w) = ((W, R, V), w)$ be a pointed Kripke model. For any $q \in At \cup H \cup HH, a \in Ag, \varphi \in \mathcal{L}$ and any multi-pointed event model $\mathcal{E}$, satisfaction of $\mathcal{L}$-formulas in $(\mathcal{M}, w)$ is given by the following clauses extended with the standard clauses for the propositional connectives:

$$\begin{array}{lll} (\mathcal{M}, w) \vDash q & \text{iff} & q \in V(w); \\ (\mathcal{M}, w) \vDash B_a\varphi & \text{iff} & (\mathcal{M}, v) \vDash \varphi \text{ for all } (w, v) \in R_a; \\ (\mathcal{M}, w) \vDash [\mathcal{E}]\varphi & \text{iff} & \text{if } \mathcal{E} \text{ is applicable in } (\mathcal{M}, w) \text{ then} \\ & & (\mathcal{M}, w) \otimes \mathcal{E} \vDash \varphi. \end{array}$$

We say that a formula φ is *valid* if $(\mathcal{M}, w) \vDash \varphi$ for all pointed Kripke models $(\mathcal{M}, w)$, and in that case we write $\vDash \varphi$.

Event Model for Attention to Attention In this section, we introduce a specific event model where agents may pay attention to whether other agents are paying attention to some subsets of the announced formula. The model is a generalization of the model presented in [4], so we keep the assumptions they make in that paper. To illustrate, the model captures the announcement (or revelation) of a conjunction of literals $(\neg)p_1 \wedge \cdots \wedge (\neg)p_n$, which are interpreted as the parallel exposure of agents to multiple stimuli. For example, it could be that we are at a talk and the speaker is presenting a theorem t and a corollary c. This corresponds to the "announcement" or revelation of $t \wedge c$. The model below also contains all logical combinations of positive and negative literals formed with t and c. This is because, as mentioned above, we want to account for an agent a learning about another agent b learning *whether* an announcement (conjunction) is true or false, and in the case in which it is false, then a should think possible that b learns any logical combination of the literals in the conjunction.

The event models presented below are composed by events which are here all represented by conjunctive formulas. Each formula is the event's own preconditions. They specify, for each subset of the set of announced literals, whether each agent is paying attention to it or not, as well as whether each agent is paying attention to other agents paying attention to it or not. The relations between events are given by a set of *edge principles*, which describe concisely what agents consider possible and learn, given their attention profiles.

Definition 29 (Attention to Attention Event Model $\mathcal{F}(\varphi)$)**.** Let $\varphi = \ell(p_1) \wedge \cdots \wedge \ell(p_n) \in \mathcal{L}$, where for each p_i, either $\ell(p_i) = p_i$ or $\ell(p_i) = \neg p_i$.

The multi-pointed event model $\mathcal{F}(\varphi) = ((E, Q, id_E), E_d)$ is defined by:

$$E = \{ \bigwedge_{p \in S} (\bigwedge_{p \in X} \ell(p) \wedge \bigwedge_{p \in S \setminus X} \neg \ell(p)) \wedge \bigwedge_{a \in Ag} (\bigwedge_{p \in Y_a} \mathsf{h}_a p \wedge \bigwedge_{p \in S \setminus Y_a} \neg \mathsf{h}_a p) \wedge \bigwedge_{(a,b) \in Ag^2} (\bigwedge_{p \in Z_{ab}} \mathsf{h}_a \mathsf{h}_b p \wedge \bigwedge_{p \in S \setminus Z_{ab}} \neg \mathsf{h}_a \mathsf{h}_b p) \\ : S \subseteq At(\varphi), X \subseteq S, \textit{ for all } a \in Ag, Y_a \subseteq S, \textit{ for all } (a, b) \in Ag^2, Z_{ab} \subseteq S \}$$

Q_a is such that $(e, f) \in Q_a$ iff all the following hold for all p:

- ATTENTIVENESS1: if $\mathsf{h}_a p \in e$, then if $(\neg)\ell(p) \in e$ then $\ell(p), \mathsf{h}_a p \in f$;
- ATTENTIVENESS2: if $\mathsf{h}_a \mathsf{h}_b p \in e$, then if $(\neg)\mathsf{h}_b p \in e$ then $(\neg)\mathsf{h}_b p, \mathsf{h}_a \mathsf{h}_b p \in f$;
- INERTIA1: if $\mathsf{h}_a p \notin e$ then $\ell(p), \neg\ell(p) \in f$;
- INERTIA2: if $\mathsf{h}_a \mathsf{h}_b p \notin e$ then $\mathsf{h}_b p, \neg\mathsf{h}_b p \in f$;

$$E_d = \{\psi \in E \colon \ell(p) \in \psi, \text{ for all } \ell(p) \in \varphi\}.$$

Current work

We are currently working on generalizing the attention to attention event model, so that it is able to account for any number of nesting of attention atoms, as for example $\mathsf{h}_a \mathsf{h}_b \mathsf{h}_a p$. This sort of higher-order attention expressions are relevant to capture what the literature calls *attentional contact*, and thus it will be central when analysing and characterizing common knowledge in a finitary way.

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Notational Variance in Substructural Logics

Camillo Fiore

11:00–11:30

University of Buenos Aires / IIF-SADAF-CONICET

For our purposes, a *logical system* is a pair $\langle \mathcal{L}, \Rightarrow \rangle$ where $\mathcal{L}$ is a formal language and $\Rightarrow$ is a dyadic relation standing for logical consequence in that language. There are lots of logical systems in the market. Often, however, two or more of those systems differ in a merely superficial way: they differ only in the symbols for a given constant (e.g. '$\wedge$' vs. '&' for conjunction), or in the syntactic conventions (e.g. Russellian vs. Polish notation), or in the primitive constants (e.g. negation and conjunction vs. negation and disjunction). When two systems differ only in such a superficial way, we say that they are *notational variants* of one another.

In the last years, we have witnessed the emergence of various logics that we may call 'radically substructural'. Traditionally, investigations in substructural logics focused mostly on properties such as contraction, exchange, monotonicity or associativity (see, e.g. [15]). The logics we have in mind, in contrast, challenge the core of the Tarskian conception of logical consequence by abandoning the properties of reflexivity and/or transitivity. Systems of this sort can serve various purposes, but their most recent popularisation has to do with the non-trivial treatment of various paradoxical phenomena (see [1] for a survey).

The guiding question of this paper is: what conditions are necessary and sufficient for two logical systems to be notational variants? First, I will argue that radically substructural logics pose serious challenges to our extant answers to these questions; in short, our usual criteria of notational variance either under-generate or over-generate in presence of these logics. Second, I will propose new criteria which overcome these challenges.

I start the paper by laying down what I take to be the standard approach to notational variance (exemplified by [12, 13, 16]). It relies on a central idea, namely, that two logical systems are notational variants just in case they are what we call 'coextensive modulo translation'. More precisely, let $\mathbf{L}_1 = \langle \mathcal{L}_1, \Rightarrow_1 \rangle$ and $\mathbf{L}_2 = \langle \mathcal{L}_2, \Rightarrow_2 \rangle$ be logical systems. The standard approach is that these systems are notational variants just in case there is pair of translations τ_1 and τ_2 such that (i) τ_1 faithfully embeds $\mathbf{L}_1$ in $\mathbf{L}_2$, (ii) τ_2 faithfully embeds $\mathbf{L}_2$ in $\mathbf{L}_1$, and in addition (iii) the following 'inversion' requirement is fulfilled:

$$A \Leftrightarrow_1 \tau_2(\tau_1(A)) \qquad\qquad A \Leftrightarrow_2 \tau_1(\tau_2(A))$$

The purpose of this last requirement is to guarantee that the two translations are, so to speak, mutually coherent.[2]

In the first substantive part of the paper I analyse how non-reflexive logics pose a challenge to the standard approach. I take logic **TS** (see [11]) as my test-case. I present it as the system $\langle \mathcal{L}, \Rightarrow_{\mathbf{TS}} \rangle$, where $\mathcal{L}$ is a propositional language with constants $\neg$, $\vee$ and $\wedge$, and relation $\Rightarrow_{\mathbf{TS}}$ is defined using the strong Kleene valuations. The sense in which **TS** undermines the standard approach is straightforward: its non-reflexivity will always induce a failure of the inversion requirement. To illustrate, let τ_1 and τ_2 be two copies of the identity function on $\mathcal{L}$. Clearly, τ_1 and τ_2 faithfully embed **TS** into itself. However, $p \not\Leftrightarrow_{\mathbf{TS}} \tau_2(\tau_1(p))$. So, the standard approach under-generates: it delivers that some systems are not notational variants of themselves. After considering various alternatives, I propose to amend the standard approach by replacing the original inversion requirement by the following one:[3]

$$\begin{aligned} A, \Gamma \Rightarrow_1 C \quad &\text{iff} \quad \tau_2(\tau_1(A)), \Gamma \Rightarrow_1 C \\ \Gamma \Rightarrow_1 C \quad &\text{iff} \quad \Gamma \Rightarrow_1 \tau_2(\tau_1(C)) \end{aligned}$$

[2] Typically, some additional syntactic constraints are imposed on τ_1 and τ_2—for instance, that they be the identity function for atomic formulas. But for our current purposes we can bypass those constraints.

[3] I draw inspiration from Belnap's [5] definition of uniqueness for connectives.

$$
\begin{array}{rcl}
A, \Gamma \Rightarrow_2 C & \text{iff} & \tau_1(\tau_2(A)), \Gamma \Rightarrow_2 C \\
\Gamma \Rightarrow_2 C & \text{iff} & \Gamma \Rightarrow_2 \tau_1(\tau_2(C))
\end{array}
$$

The resulting criterion solves the problem observed, rendering **TS** a notational variant of itself. Also, it delivers the usual verdicts in the more familiar cases. Thus, it constitutes an improvement over the standard approach.

In the second substantive part of the paper I analyse how non-transitive logics pose a challenge even to this last, amended criterion. I take logic **ST** (see [6]) as my test-case. I present it as the system $\langle \mathcal{L}, \Rightarrow_{\mathbf{ST}} \rangle$, where $\mathcal{L}$ is as before and $\Rightarrow_{\mathbf{ST}}$ is also defined using the strong Kleene valuations. Furthermore, I present classical logic **CL** as the system $\langle \mathcal{L}, \Rightarrow_{\mathbf{CL}} \rangle$, where $\Rightarrow_{\mathbf{CL}}$ is defined as usual using the Boolean bivaluations. The sense in which **ST** undermines our amended criterion is the following. On the one hand, **ST** and **CL** have exactly the same valid inferences; as a consequence, they are trivially declared notational variants. On the other hand, however, **ST** supports naive, non-trivial theories of paradoxical phenomena which trivialise **CL**; this has been considered by many authors (e.g. [3, 7, 9]) as a sufficient reason to say that **ST** and **CL** are not mere notational variants. So, the amended criterion over-generates: it declares as notational variants systems that are intuitively not. I consider some possible solutions to this problem; in particular, the proposal that emerges from the works of Barrio et. al. [2, 4], according to which two logical systems are notational variants just in case they are coextensive modulo translation not only in their valid inferences, but also in their valid meta-inferences of any finite level. I claim that even this refined criterion over-generates claims of notational variance. Then, I move on to present my alternative solution. Intuitively, I say that two logical systems are notational variants just in case the *non-logical theories* that they support are coextensive modulo translation.[4] Moreover, and crucially, I propose to understand non-logical theories not as collections of *formulas*, but as collections of *inferences*. To put things a bit more formally: Let $\mathbf{L} = \langle \mathcal{L}, \Rightarrow \rangle$ be a logical system, and let $\mathbb{T}$ be a set of inferences on $\mathcal{L}$, where an inference on $\mathcal{L}$ is a pair of collections of $\mathcal{L}$ formulas. We will write $\mathbf{L}^{\mathbb{T}} = \langle \mathcal{L}, \Rightarrow^{\mathbb{T}} \rangle$ to denote the formal system that results from adding all inferences in $\mathbb{T}$ to $\mathbf{L}$. Relation $\Rightarrow^{\mathbb{T}}$ might be obtained in different ways. For instance,

- If $\Rightarrow$ is given by model-theoretic means, then we could define $\Rightarrow^{\mathbb{T}}$ by restricting the models of $\Rightarrow$ to those that satisfy each inference in $\mathbb{T}$.
- If $\Rightarrow$ is given by means of a sequent calculus $\mathcal{S}$, we could arrive at $\Rightarrow^{\mathbb{T}}$ by adding the inferences in $\mathbb{T}$ to $\mathcal{S}$ as axioms.

No matter how relation $\Rightarrow^{\mathbb{T}}$ is induced, our reading of $\mathbf{L}^{\mathbb{T}}$ will be the same: it is the non-logical theory given by the set of inferences $\mathbb{T}$ over the underlying logic $\mathbf{L}$. Then, our criterion can be formulated as follows: Two logical systems $\mathbf{L}_1 = \langle \mathcal{L}_1, \Rightarrow_1 \rangle$ and $\mathbf{L}_2 = \langle \mathcal{L}_2, \Rightarrow_2 \rangle$ are notational variants just in case there is a pair of translations τ_1 and τ_2 from $\mathcal{L}_1$ to $\mathcal{L}_2$ and viceversa such that: (i) For every set of inferences $\mathbb{T}$ on $\mathcal{L}_1$, τ_1 and τ_2 render coextensive modulo translation the formal systems $\mathbf{L}_1^{\mathbb{T}}$ and $\mathbf{L}_2^{\tau_1(\mathbb{T})}$, and (ii) For every set of inferences $\mathbb{S}$ on $\mathcal{L}_2$, τ_1 and τ_2 render coextensive modulo translation the formal systems $\mathbf{L}_2^{\mathbb{S}}$ and $\mathbf{L}_1^{\tau_2(\mathbb{S})}$. This new criterion successfully differentiates between systems **ST** and **CL**. Moreover, it also gives the right answers in the more familiar cases. Lastly, I claim that in the literature we find independent motivation to understand theories as collections of inferences rather than collections of formulas; for instance, such a liberal understanding of theories is a prerequisite for the treatment of paradoxical phenomena with systems such as **K3** or **TS** (see [10] and [14], respectively, for recent examples). For all these reasons, I conclude that the new criterion proposed constitutes a substantial improvement over our usual criteria of notational variance; in particular, it still gives us correct answers when dealing with radically substructural logical systems.

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Reading Newton's *De Analysi* by hyperfinite sums

Piotr Błaszczyk

11:30–12:00

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In 1669 manuscript *De Analysi*, Newton derives three theorems, now cornerstones of modern calculus: the power series for arcsine, the power series for sine, and that the area under the curve $y(x) = x^{\frac{m}{n}}$ equals $\frac{n}{m+n} x^{\frac{m+n}{n}}$ (Rule I). He also sets the rule stating that the area under finitely or infinitely many curves equals the sum of areas under each curve (Rule II) and shows how to expand into a power series of then-standard functions such as $\frac{a^2}{b+x}$ or $\sqrt{a^2+x^2}$ base on some algebraic laws (Rule III).

Proving Rule I, Newton's approach hinges on an odd procedure of summing up infinitesimal area moments; for the series of arcsine, he combines Rule II applied to infinitesimal arc moments with the same peculiar sum operation; dealing with area and arc moments, he employs the concept infinitely small unit segment, *indivisible*; deriving the sine series, he adopts an approach other than Rules I to III.

The standard interpretation of *De Analysi* relies on the Riemann integral and takes Rule I as the fundamental theorem of calculus: $(\int_0^x f dt)' = f(x)$; it connects Rule II to the theorem on the integration of infinite series: $\int_0^x (\sum_1^\infty f_n) dt = \sum_1^\infty (\int_0^x f_n dt)$; and interprets Rule III as Taylor's series theorem. That interpretation, first, does not correspond to the arguments' structure regarding the series of arcsine and sine: unlike the modern approach, Newton derived the arcsine series before the sine series; second, the standard proof of Rule I requires properties of the exponential function to determine the derivative of $f(x) = x^{\frac{m}{n}}$, which make it anachronic; third, for Newton, finite and infinite cases in Rule II do not require separate arguments, while under the standard interpretation, these cases are substantially different.

We interpret *De Analysi* with techniques of nonstandard analysis and represent Newton's arguments on a hyperfinite grid, essentially a discrete domain instead of a continuous one. Bridging the gap between finite and infinite, we mimic Newton's approach and define the area under a curve as a hyperfinite sum. We provide rigorous proof of Newton's Rules I and II and describe a possible process of discovering Rule I using 17th-century techniques.

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Hypersequent calculi for propositional default logics

Mario Piazza, Andrea Sabatini

11:30–12:00

Scuola Normale Superiore di Pisa

Default reasoning is a form of non-monotonic reasoning that allows us to draw plausible conclusions under incomplete information and in absence of explicit evidence to the contrary: when disproved by new evidence, these conclusions can be withdrawn [20, 13, 2]. From a logical perspective, default reasoning can be formalized by extending classical logic with a collection of *extra-logical axioms* – which encompass the propositional contents of beliefs held by an ideal reasoner – along with a set of *default rules* encapsulating the informational pathways she follows to arrive at defeasible conclusions.

Starting with Reiter's work [20], default logic's formalism has first been developed by employing Hilbert-style calculi [12, 10, 6, 8], and subsequently investigated through semantic tableaux [1, 21] as well as sequent calculi [5, 9]. However, these proof-theoretic methods have undesirable features on different fronts. Very sketchy:

(*i*) the proof-search space in Hilbert-style calculi is infinite: this feature makes these calculi less suitable for representing complex reasoning tasks;

(*ii*) in tableaux-based calculi, the information flow cannot be controlled at the local level, making it challenging to enforce fine-grained constraints;

(*iii*) sequent calculi, while versatile, rely on *ad hoc* extensions of the underlying language: this feature makes them less suitable for a *modular* and *uniform* proof-theoretic treatment encompassing axiomatic extensions of classical logic [14].

The aim of this talk is to introduce a novel proof-theoretic approach to default propositional logics, centered on a non-standard notion of *hypersequent*. Traditionally, hypersequents are lists of sequents separated by a bar, originally conceived to provide analytic calculi for modal and intermediate logics lacking cut-free sequent calculi [3, 4]. We modify the notion of hypersequent in order to embed within derivation trees the consistency checks involved in the application of default rules: specifically, we redefine hypersequents as *hybrid* constructs, each comprising a sequent and a *set* of *antisequents*. Departing from the conventional disjunctive interpretation of the separating bar, we embrace a conjunctive reading [19, 17]. In this framework, antisequents within a hybrid hypersequent furnish contrary updates concerning the provability of the associated sequent.

Our formalization of default rules through hybrid hypersequents involves specifying distinct *extra-logical rules*. Here is the general idea. Initially, we convert each default's prerequisite into conjunctive normal form while translating its conclusion into clausal form. Subsequently, for every default rule and each clause within its prerequisite, we generate two hybrid hypersequents: the first incorporates the clause into its provability part, whereas the second features the (possibly weakened) combination of the clause with the conclusion in its provability part. These hybrid hypersequents function as premises for the extra-logical rule corresponding to the default. The conclusion of such rule comprises a hybrid hypersequent, whose refutability part consists of all antisequents in the premises alongside the set of negated justifications. This method yields a Gentzen-style formulation of defaults, characterized by constrained instances of *Strengthening* – essentially, the inverse rule of Weakening [7].

On this basis, we design hybrid hypersequent HG4 calculi for default logics. We claim that these calculi overcome some drawbacks of previous formalisms:

(*i*) the proof-search space in HG4 calculi is finite;

(*ii*) HG4 calculi allow for the local control of the information flow;

(*iii*) HG4 calculi do not rely on *ad hoc* extensions of the underlying language.

First, we introduce the formal apparatus for handling axiomatic extensions of classical logic, namely hybrid sequent calculi with crucial proof-theoretic properties [18]. Next, our focus shifts to hybrid hypersequent calculi. Here, we establish admissibility of structural rules, invertibility of logical rules and a weakened version of the subformula property for cut-free proofs. We present hybrid hypersequent calculi that are sound and (weakly) complete with respect to credulous consequence based over *Łukaszewicz extensions* [12], showing that they fail to be strongly complete due to their non-monotonic behaviour in relation to the addition of extra-logical axioms. Moreover, we highlight that admissible rules fail to be encoded in provable hypersequents because of the context-sensitivity of extra-logical rules. Subsequently, we present a hypersequent-based decision procedure for skeptical consequence: this method relies on the *abductive* search of counterexamples, thereby circumventing the need for early computation of all extensions [16]. Lastly, we outline avenues for future research, with a brief discussion on hybrid hypersequent calculi for credulous consequence based on *Reiter extensions* [20, 22] and *exclusionary default reasoning* [11], as well as strongly complete, *controlled sequent* calculi for modified credulous consequence [15].

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Natural deduction for definite descriptions in strong Kleene free logic

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14:00–14:30

Funded by the European Union (ERC, ExtenDD, project number: 101054714). Views and opinions expressed are however those of the author only and do not necessarily reflect those of the European Union or the European Research Council. Neither the European Union nor the granting authority can be held responsible for them.

In this talk, we introduce a natural deduction system for a neutral free logic based on the three-valued strong Kleene logic extended by definite descriptions. Due to the third truth value formulae with non-denoting terms have no classical truth value. We define definite descriptions, following Lambert's axiom, which yields the minimal theory of definite descriptions.

Free logics are known for their capacity to consider singular terms that are not assumed to denote an existing object, while quantifiers are presumed to have existential support. There are several approaches to analysing atomic formulae with non-denoting terms. Positive free logic allows them to be true; negative free logic requires them to always be false; and neutral free logic assigns them the third value, which can be interpreted as 'undefined' or 'neither true nor false'.

The interpretation of definite descriptions in free logics often relies on the Lambert axiom:

$$\forall y(y = \imath x\varphi \leftrightarrow \forall x(\varphi \leftrightarrow y = x)) \tag{L}$$

where $\imath x\varphi$ is closed and does not have any occurrence of y. Moreover, for simplicity, we impose the following limitation: the expression $\imath x\varphi$ does not have any other definite descriptions inside it. This approach leads to the minimal theory of definite descriptions.

There are many recent publications that focus on the proof theory for free logics and free logics with definite descriptions. Considering the space constraints, we will only mention a few relevant papers. Woodruff's paper [5] focuses on natural deduction systems for neutral free logics. Pavlović and Gratzl's paper [4] explores non-standard sequent calculi for these logics. Indrzejczak's paper [1] discusses sequent calculi for positive and negative free logics with definite descriptions. Additionally, there is another paper [2] that addresses similar issues for neutral free logics with definite descriptions.

This study aims to provide a natural deduction version of the findings from [2] that extend the research conducted by Pavlović and Gratzl in their paper [4] by identity and definite descriptions. Pavlović and Gratzl see strong Kleene logic $\mathbf{K}_3$ and weak Kleene logic $\mathbf{K}_3^w$ [3] as a basis for neutral free logic. For the purpose of simplicity and due to space constraints, we will solely concentrate on the logic $\mathbf{K}_3$. We present a Gentzen-Prawitz-style natural deduction system for the propositional fragment of $\mathbf{K}_3$ in the language with the connectives $\neg, \wedge, \vee, \rightarrow$. We then extend the system by incorporating the universal quantifier $\forall$, the rules that define the existence predicate $\mathcal{E}$, the identity predicate $=$, and finally the $\imath$-term forming operator that handles definite descriptions.

The propositional rules for $\mathbf{K}_3$ are as follows:

$$(\vee I_1)\ \frac{A}{A \vee B} \quad (\vee I_2)\ \frac{B}{A \vee B} \quad (\rightarrow I_1)\ \frac{\neg A}{A \rightarrow B} \quad (\rightarrow I_2)\ \frac{B}{A \rightarrow B} \quad (\wedge I)\ \frac{A \quad B}{A \wedge B}$$

$$(\vee E)^{i,j}\ \frac{A \vee B \quad \begin{matrix}[A]^i \\ \mathfrak{D}_1 \\ C\end{matrix} \quad \begin{matrix}[B]^j \\ \mathfrak{D}_2 \\ C\end{matrix}}{C} \qquad (\rightarrow E)\ \frac{A \rightarrow B \quad \begin{matrix}[\neg A] \\ \Pi_1 \\ C\end{matrix} \quad \begin{matrix}[B] \\ \Pi_2 \\ C\end{matrix}}{C}$$

$$(\wedge E_1)\ \frac{A \wedge B}{A} \qquad (\wedge E_2)\ \frac{A \wedge B}{B} \qquad (\mathrm{EFQ})\ \frac{\neg A \quad A}{B} \qquad (\neg\neg I)\ \frac{A}{\neg\neg A} \qquad (\neg\neg E)\ \frac{\neg\neg A}{A}$$

$$(\neg{\to}I)\ \frac{A \quad \neg B}{\neg(A \to B)} \qquad (\neg{\to}E_1)\ \frac{\neg(A \to B)}{A} \qquad (\neg{\to}E_2)\ \frac{\neg(A \to B)}{\neg B}$$

$$(\neg\vee I)\ \frac{\neg A \quad \neg B}{\neg(A \vee B)} \qquad (\neg\vee E_1)\ \frac{\neg(A \vee B)}{\neg A} \qquad (\neg\vee E_2)\ \frac{\neg(A \vee B)}{\neg B}$$

$$(\neg\wedge I_1)\ \frac{\neg A}{\neg(A \wedge B)} \qquad (\neg\wedge I_2)\ \frac{\neg B}{\neg(A \wedge B)} \qquad (\neg\wedge E)^{i,j}\ \dfrac{\neg(A \wedge B) \quad \begin{array}{c}[\neg A]^i\\ \mathfrak{D}_1\\ C\end{array} \quad \begin{array}{c}[\neg B]^j\\ \mathfrak{D}_2\\ C\end{array}}{C}$$

The rules for $\forall$ are as follows:

$$(\forall I)^i\ \dfrac{\begin{array}{c}[\mathcal{E}a]^i\\ \mathfrak{D}\\ A^x_a\end{array}}{\forall x A} \qquad (\forall E)\ \frac{\forall x A \quad \mathcal{E}t}{A^x_t} \qquad (\neg\forall I)\ \frac{\neg A^x_t \quad \mathcal{E}t}{\neg\forall x A} \qquad (\neg\forall E)^{i,j}\ \dfrac{\neg\forall x A \quad \begin{array}{c}[\neg A^x_a]^i, [\mathcal{E}a]^j\\ \mathfrak{E}\\ C\end{array}}{C}$$

where in $(\forall I)$, the parameter a does not occur free in any undischarged assumptions of $\mathfrak{D}$ except $\mathcal{E}a$; in $(\forall E)$ and $(\neg\forall I)$, t is free for x in A; in $(\neg\forall E)$, the parameter a does not occur in $\neg\forall x A(x)$, nor in C, nor in any formulas undischarged in $\mathfrak{E}$ except $\neg A^x_a$ and $\mathcal{E}a$.

The rules for $\mathcal{E}$ are given below, where $P(\vec{t})$ stands for $P(t_1, \ldots, t_n)$:

$$(\mathrm{EM}_{\mathcal{E}})^{i,j}\ \dfrac{\begin{array}{c}[\mathcal{E}t]^i\\ \mathfrak{D}_1\\ C\end{array} \quad \begin{array}{c}[\neg\mathcal{E}t]^j\\ \mathfrak{D}_2\\ C\end{array}}{C} \qquad (P[t]E)^i\ \dfrac{P[t] \quad \begin{array}{c}[\mathcal{E}t]^i\\ \mathfrak{D}\\ C\end{array}}{C} \qquad (\neg P[t]E)^i\ \dfrac{\neg P[t] \quad \begin{array}{c}[\mathcal{E}t]^i\\ \mathfrak{D}\\ C\end{array}}{C}$$

$$(\neg P(\vec{t})I)^{i,j}\ \dfrac{\begin{array}{c}[\mathcal{E}t_1, \ldots, \mathcal{E}t_n, P(\vec{t})]^i\\ \mathfrak{D}_1\\ C\end{array} \quad \begin{array}{c}[\neg P(\vec{t})]^j\\ \mathfrak{D}_2\\ C\end{array}}{C} \qquad (P(\vec{t})I)^{i,j}\ \dfrac{\begin{array}{c}[\mathcal{E}t_1, \ldots, \mathcal{E}t_n, \neg P(\vec{t})]^i\\ \mathfrak{D}_1\\ C\end{array} \quad \begin{array}{c}[P(\vec{t})]^j\\ \mathfrak{D}_2\\ C\end{array}}{C}$$

where both $P[t]$ and $P(t_1, \ldots, t_n)$ denote atoms or identities but not $\mathcal{E}t$, moreover identities of the form $b = d$ are excluded. In $P[t]$ there is at least one occurrence of t and there may be other terms; in $P(t_1, \ldots, t_n)$ there are no other terms.

The rules for $=$ are as follows:

$$(\neg = I)^{i,j,k}\ \dfrac{\begin{array}{c}[\neg t \approx s]^i\\ \mathfrak{D}_1\\ C\end{array} \quad \begin{array}{c}[\neg A^x_t]^j\\ \mathfrak{D}_2\\ C\end{array} \quad \begin{array}{c}[A^x_s]^k\\ \mathfrak{D}_3\\ C\end{array}}{C} \qquad (= I_1)^i\ \dfrac{\mathcal{E}t \quad \begin{array}{c}[t = t]^i\\ \mathfrak{D}\\ C\end{array}}{C} \qquad (= I_2)^i\ \dfrac{\mathcal{E}d \quad \begin{array}{c}[a = d]^i\\ \mathfrak{D}\\ C\end{array}}{C}$$

where A^x_t is an atomic formula, or identity, or Et, $t \approx s$ denotes either $t = s$ or $s = t$, a is a fresh parameter and d is an arbitrary description.

Finally, here are the rules for $\imath$:

$$(\imath I)^{i,j,k,l}\ \dfrac{\begin{array}{c}[c=\imath xA]^i\\ \mathfrak{D}_1\\ C\end{array}\quad\begin{array}{c}[\mathcal{E}c]^j,\\ \mathfrak{D}_2\\ A^x_c\end{array}\quad\begin{array}{c}[\mathcal{E}a]^k,[\mathcal{E}c]^k\\ \mathfrak{D}_3\\ a=c\end{array}\quad\begin{array}{c}[\mathcal{E}a]^l\\ \mathfrak{D}_4\\ \neg A^x_a\end{array}}{C}\qquad (\imath E_1)^{i}\ \dfrac{c=\imath xA\quad\begin{array}{c}[A^x_c]^i,[\mathcal{E}c]^i\\ \mathfrak{D}\\ C\end{array}}{C}$$

$$(\imath E_2)^{i,j}\ \dfrac{c=\imath xA\quad\begin{array}{c}[\mathcal{E}b]^i,[\mathcal{E}c]^i,[\neg A^x_b]^i\\ \mathfrak{D}_1\\ C\end{array}\quad\begin{array}{c}[b=c]^j,[\mathcal{E}b]^j,[\mathcal{E}c]^j\\ \mathfrak{D}_2\\ C\end{array}}{C}$$

$$(\neg\imath I_1)^{i,j}\ \dfrac{\begin{array}{c}[\neg c=\imath xA]^i\\ \mathfrak{D}_1\\ C\end{array}\quad\begin{array}{c}[\mathcal{E}c]^j\\ \mathfrak{D}_2\\ \neg A^x_c\end{array}}{C}\qquad(\neg\imath I_2)^{i,j,k}\ \dfrac{\begin{array}{c}[\neg c=\imath xA]^i\\ \mathfrak{D}_1\\ C\end{array}\quad\begin{array}{c}[\mathcal{E}b]^j\\ \mathfrak{D}_2\\ A^x_b\end{array}\quad\begin{array}{c}[\mathcal{E}b]^k,[\mathcal{E}c]^k\\ \mathfrak{D}_3\\ \neg b=c\end{array}}{C}$$

$$(\neg\imath E)^{i,j}\ \dfrac{\neg c=\imath xA\quad\begin{array}{c}[\mathcal{E}c]^i,[\neg A^x_c]^i\\ \mathfrak{D}_1\\ C\end{array}\quad\begin{array}{c}[\mathcal{E}a]^j,[\mathcal{E}c]^j,[\neg a=c]^j,[A^x_a]^j\\ \mathfrak{D}_2\\ C\end{array}}{C}$$

where in $(\imath I)$, the parameter a does not occur in any undischarged assumption of $\mathfrak{D}_3$ and $\mathfrak{D}_4$, except $\mathcal{E}a$; in $(\neg\imath E)$, the parameter a does not occur in any undischarged assumption of $\mathfrak{D}_2$, except $\mathcal{E}a$, $\neg a=c$, and A^x_a.

During our talk we plan to introduce this natural deduction system and describe its properties. In particular, we explore the possibility of proving the normalisation theorem and establishing the negation subformula property. Besides, we intend to describe an adequate semantics for this natural deduction system as well as its connection with the sequent-based calculi from [4] and [2].

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General Tableaux Method for Metainferential Logics

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14:00–14:30

Traditionally, a logic was equated to the set of inferences it deems valid. However, recent developments in the field of substructural logics have posed a challenge to this idea. More specifically, the mixed logic **ST**, defended by Cobreros, Egré, Ripley and van Rooij as a solution to both the sorites paradox ([4]) and the semantic paradoxes ([5], [12]), has shown a way in which the traditional way of characterizing a logic falls short: **ST** has the same valid inferences as **CL** whilst being non-transitive, in the sense that *Cut* is locally invalid according to its semantics. In spite of both sharing the same set of valid inferences, **ST** would appear to be a non-classical logic, given that the transitivity of the logical consequence relation (codified in the *Cut* rule) is usually regarded as a key feature of classical reasoning.

In light of this problem, and bearing in mind that *Cut* is not an inference between formulae, but rather an inference between inferences (*i.e.*, a metainference), several authors ([2], [1]) shifted the focus from inferences to metainferences, and defended that logics are not only defined by their inferences, but also by their metainferences. According to this view, logics that coincide in their inferences, but not in their metainferences, are considered different. This kick-started the development of what we now call *metainferential logics*: logics where we have metainferences of any level n, which are built as hierarchies over known inferential logics.

Before laying down our proposal, let us first introduce some indispensable technical machinery regarding inferences. We will be working with a propositional language $\mathcal{L}$. We use uppercase latin letters $(A, B, C...)$ as stand-ins for formulas of any complexity. Let $\mathcal{V}$ be a set of truth-values. Following [3], we stipulate that a standard X is a non empty subset of $\mathcal{V}$. We say that a valuation v satisfies a formula A according to the standard X if and only if $v(A) \in X$.

An inference on $\mathcal{L}$ is an ordered pair of sets of sentences of $\mathcal{L}$, written $\Sigma \Rightarrow \Pi$. We say that a valuation v confirms or satisfies an inference $\Sigma \Rightarrow \Pi$ according to a pair of standards XY if and only if, if v satisfies every $A \in \Sigma$ according to the standard X, then v satisfies some $B \in \Pi$ according to the standard Y. On the contrary, call a valuation v that satisfies every premise according to X but doesn't satisfy any conclusion according to Y a *counterexample* in XY to said inference. Crucially, X and Y need not be the same.

Notice that a set of valuations V and a pair of standards XY is enough to define a (mixed) consequence relation, which in turn, can be used to characterize an (inferential) logic **XY**: an inference $\Sigma \Rightarrow \Pi$ is valid in **XY** if and only if every valuation satisfies it according to XY. Equivalently, if and only if no v is a counterexample in XY to it.

Following [3], we say that *mixed* logics are those whose consequence relation can be characterized in this manner. When the standard for the premises and the standard for the conclusions coincide, we call them mixed and *pure* logics. When they are not the same, we say they are mixed and *impure* logics.

This concludes the preliminaries about inferential logics. For metainferential logics, we proceed in a similar fashion. We define a metainference (of level 1) as an ordered pair of sets of inferences, written $\Gamma \Rightarrow_1 \Delta$.

The notion of metainferential validity can be made precise in two ways: as preservation of satisfaction (local validity) or as preservation of validity (global validity). The first view is the most wide-spread and the one we will be working with. We say that a metainference (of level 1) is locally valid in the metainferential logic **XY/WZ** if and only if, for every valuation v, if v satisfies each inference $\gamma \in \Gamma$ according to the pair of standards XY, then v satisfies some inference $\delta \in \Delta$ according to the pair of standards WZ. Again, notice that inferences in Γ and inferences in Δ could, in principle, be evaluated under different inferential standards, thus obtaining metainferential mixed impure logics.

This construction can be iterated to obtain metainferences of any level n, for $n \geq 1$, defined as ordered pairs of sets of metainferences of level $n-1$, and written $\Gamma \Rightarrow_n \Delta$.

Accordingly, a metainferential logic can be of any finite level n, since we can also define a consequence relation that specifies preservation of satisfaction over a pair of standards for metainferences of level n (where $n < \omega$): a metainference (of level $n \geq 1$) is locally valid in the metainferential logic **XY** if and only if, for every valuation v, if v satisfies each metainference (of level $n-1$) $\gamma \in \Gamma$ according to the pair of standards found in X, then v satisfies some metainference (of level $n-1$) $\delta \in \Delta$ according to the pair of standards found in Y.

So far, we made a brief introduction to metainferential logics from a semantic perspective. Some proof-theoretic accounts have been presented in the literature, although most of them either concentrate on level 1 metainferences ([7]) or are limited to the Strong Kleene family of metainferential logics ([8], [6], [9], [10]). The main goal of this talk is to remedy this lack of generality, by providing a method to build tableaux for metainferential logics (of any level n) upon tableaux for inferential logics. The general recipe we provide here helps us obtain metainferential tableaux for metainferential logics based on the Strong or Weak Kleene three-valued schema, the Belnap-Dunn four-valued schema, and in principle, infinitely many others, as long they are finitely valued and meet certain conditions.

To achieve our aim, we work with three types of rules: general rules for metainferences, general rules for formulas, and lastly, particular rules for the connectives (of a given valuation schema). The first two kinds of rules (general rules) can be added on top of different sets of particular rules, thus creating a metainferential tableaux system for metainferential logics which, in turn, use different inferential logics as building-blocks. We consider this 'modular' quality of our tableaux rules to be the main advantage of the proof system we present here, and what sets it apart from the rest of the proof systems for metainferential logics found in the literature.

To demonstrate how our tableaux work, we take as an example the Strong Kleene semantics, which has three truth-values: $\{1, \frac{1}{2}, 0\}$. We define two standars, $s = \{1\}$ and $t = \{1, \frac{1}{2}\}$. With these, we can define four very well-known inferential logics: **K3**, whose consequence relation is defined for the pair of standards SS; **LP**, whose consequence relation is defined for the pair TT; **TS**, whose consequence relation is defined for the pair TS; and lastly, **ST**, whose consequence relation is defined for the pair ST. One can take these 4 inferential consequence relations as metainferential standards, thus obtaining 16 new metainferential consequence relations of level 1 (as witnessed by [11]). And by iterating this procedure, one can generate the Strong Kleene hierarchy of metainferential logics.

We work with two kinds of labels: γ, x and $\gamma, \overline{x}$. The former can be interpreted as stating that γ satisfies the standard x, while the latter can be interpreted as stating that γ does not satisfy the standard x.

On the one hand, the general rules for metainferences allow us to go from inferences and metainferences of any level to the formula level:

Rules for metainferences

INF0

$$\Gamma \Rightarrow \Delta, \overline{xy}$$
$$\gamma_1, x$$
$$\vdots$$
$$\gamma_i, x$$
$$\delta_1, \overline{y}$$
$$\vdots$$
$$\delta_j, \overline{y}$$

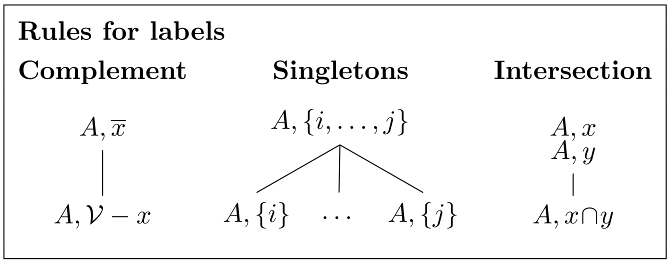
INF1

$$\Gamma \Rightarrow \Delta, xy$$
$$\gamma_1, \overline{x} \quad \ldots \quad \gamma_i, \overline{x} \quad \delta_1, y \quad \ldots \quad \delta_j, y$$

The rule *INF1* codifies (meta)inference satisfaction, since it states that if $\Gamma \Rightarrow \Delta$ is satisfied by a valuation v

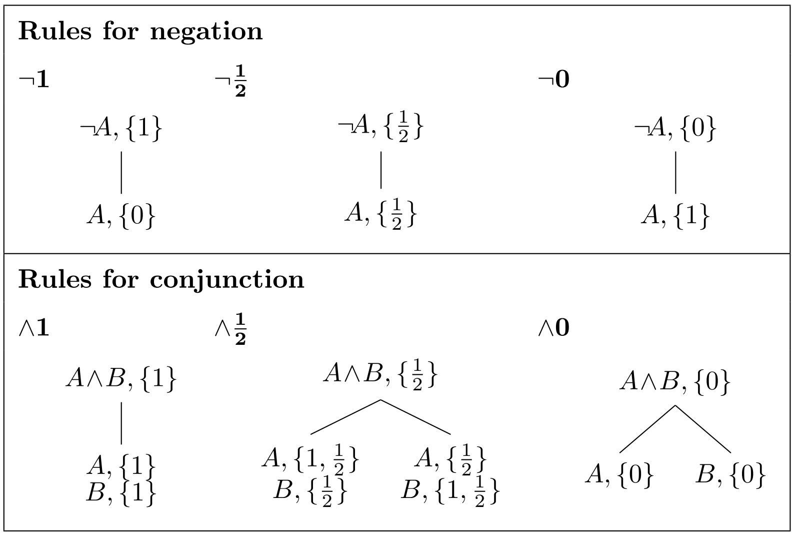
according to XY, then either some of the premises is not satisfied in v according to X or at least one of the conclusions is satisfied in v according to Y. Meanwhile, *INF0* states that if it is not the case that $\Gamma \Rightarrow \Delta$ is satisfied by a valuation according to XY, then that means every premise is satisfied in v according to X and none of the conclusions are satisfied in v according to Y. To check if a metainference $\Gamma \Rightarrow_n \Delta$ is valid in the metainferential logic **XY**, we start the tree with $\Gamma \Rightarrow \Delta, \overline{xy}$.

On the other hand, there are general rules for formulas, which allow to operate on labels:



First, *Complement* allows us to shift from a overlined label to a simple label. Secondly, *Singletons* takes a formula with a standard comprised of multiple elements (truth values $i...j$), and it opens a new branch for each element. And thirdly, *Intersection* allows us to take different nodes (on the same branch) with the formula A labelled with x and y, and extend the tree downwards with A and the label corresponding to the intersection of the standards x and y. We stipulate that a branch *closes* if and only if the intersection of A, x and A, y is empty.

Finally, the particular rules fix the inferential logic upon which we will be working on. They have to meet certain requirements, such as being finitely valued. For all operations $\star$ of $\mathcal{L}$ and every truth value $i \in \mathcal{V}$, we have to define rules of the form $\star i$, where every possible combination of operation $\star$ and truth-value i is exhausted. For instance, the rules for the Strong Kleene negation and conjunction would be this:



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Non-Deductive Term Logic Tableaux

J.-Martín Castro-Manzano

14:30–15:00

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Term Functor Logic (TFL, for short) is a relatively novel logic that follows the tradition of Aristotelian logic —hence its alternative name, Traditional Formal Logic— in the sense that it uses a term syntax rather than a Fregean syntax; however, it still needs some tweaks in order to claim its rightful place within the realm of Aristotelian logic —not that it needs to, but we would like to see it there! For instance, in other places we have offered some ways in which we can update TFL in order to comply with some criteria for relevance insofar as Aristotelian logic requires some sort of relevance. And so, following this train of thought, in this contribution we try to update TFL by adding some ways in which we can deal with non-deductive inference, namely, inductive and abductive inference, insofar as Aristotelian logic demands the treatment of non-deductive inference. Thus, in order to reach this goal, in this contribution we combine TFL and a proxy of Non-Axiomatic Logic —which is a term logic that deals with non-deductive inference— by way of a tableaux method. The result is a tableaux method within the framework of TFL that is able to model non-deductive inference.

One Problem from Carnap and Wójcicki

Rodrigo Mena Gonzalez

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Munich Center for Mathematical Philosophy

14:30–15:00

Two ideas from Carnap and Wójcicki give place to a simple problem concerning the set of logic presentations. In [1, p.29], Carnap stated that

"The choice of rules and primitive sentences — even when a definite material interpretation of the calculus is assumed beforehand — is, to a large extent, arbitrary. Often a system can be changed (without changing the content) by omitting a primitive sentence, and, in its place, laying down a rule of inference — and conversely".

A little more than fifty years later, [4] defined a *propositional logic of formulas* as a set of formulas of a propositional language closed under substitutions, and a *propositional logic of inferences* as a structural consequence operation in a propositional language. Such consequence operation is determined by one set of inferences considered valid.

It is natural to think that both types of logics are opposite ends in a possibly infinite series of presentations of the same linguistic structure, determined by the sort of variations described above by Carnap (I will call them *Carnap variations* here for lack of a better name). One obvious problem emerges from this: under what conditions would it be possible to perform this Carnap operation to logical systems to provide as many deductive bases as possible to characterize them between both Wójcicki's pure ends?

There are some natural ways to perform Carnap variations. Sometimes they seem pretty obvious, like substituting

$$\vdash (\varphi \wedge \psi) \to \varphi \tag{0.3}$$

for

$$\varphi \wedge \psi \vdash \varphi \tag{0.4}$$

After some reflection, it is unavoidable to think that every replacement of one or more inference rules should require the ascription to one or maybe more *functionally equivalent* theorems and vice versa, meaning that the set of rules of inference should provide the same results as the chosen axioms. We say that (1) and (2) are functional equivalent because thanks to (2), we can obtain the same result that we derive from (1) plus another rule, like Modus Ponens (MP):

$$\frac{\vdash \varphi \wedge \psi \quad \vdash (\varphi \wedge \psi) \to \varphi}{\vdash \varphi} MP \tag{0.5}$$

To replace (2) with (1) we should be required to add MP to still obtain the same conclusion ($\vdash \varphi$) from the same premise ($\vdash \varphi \wedge \psi$). One can object, insisting that premises and conclusions are not exactly the same in both cases. (2) is a more general rule because its application is not restricted to theorems, but it does not seem relevant when our only goal is just preserving the set of theorems of a logic after every variation of its deductive base. It is worth noting that this application of MP is necessary to choose *this precise substitution* of (2) by (1) and to provide an account of the equivalence of at least two presentations of a logic containing one of them, from a functional point of view. This also shows that replacements cannot be defined as merely one-to-one substitutions but should be considered in the context of other axioms and rules.

Functional equivalence seems essential, in any case, to preserve the same set of theorems of a logic after the variation.

Apparently, when replacing rules of inference with axioms we should also restrict our attention to one single type of rule. Following Wójcicki, we can distinguish between Hilbert and Gentzen types of rules (also H- and G-rules). Hilbert-style rules are instructions like: *´From* Γ *(being* Γ *any set of formulas including the empty set) infer* φ'. Gentzen-style rules are also instructions, but more general: *´From* Γ *infer* φ*, provided that* ψ *has been inferred from* Δ'. Note that axioms and *proper* rules of inference in a deductive base are H-rules. It seems, anyway, impossible to replace H-rules with G-rules even though standard proofs of the Deduction Theorem (DT) show that a G-rule like that could emerge from a set of H-rules. In any case, that is different from stating that DT could replace those H-rules. In any case, that is still a pending problem. Maybe, it could replace them with more G-rules. Otherwise, Gentzen-proof systems for logics originally described only by H-rules could not be possible.

The fact that DT holds for a logic is another requirement for performing Carnap's variations, that must be preserved. There are examples in the literature (like [2]) of a possible variation of a presentation of Classical Logic that is not like those suggested by Carnap. [2] offered a deductive base closing all the classical logic theorems losing the MP rule. So, even though the formula

$$(\varphi \wedge (\varphi \rightarrow \psi)) \rightarrow \psi \tag{0.6}$$

is a theorem in his system, the modus ponens rule in (2) is not derivable but only admissible. Following Wójcicki, call (4) the *generated formula* from (2).

On those systems where DT only holds partially (like in modal logics), it seems necessary to impose certain restrictions to recover it. We cannot replace a rule like ´from $\vdash \varphi$ derive $\vdash \Box\varphi$' for a formula like $\vdash \varphi \rightarrow \Box\varphi$. This suggests that the way we define DT may be important to perform Carnap's variations of a deductive base (See [3]). On systems where DT does not hold at all, the only possible variations are replacements among proper rules of inference (rules deriving formulas from a non-empty set of formulas).

One interesting case is that of replacing the last single proper rule of inference in a series of Carnap's variations. In that case, we cannot appeal to any functional equivalence given that there are no more rules that could grant such equivalence (as MP did in our example above). The replacement of the last rule should give place to the whole set of theorems of a logic. This is natural to conclude in the case of MP from the fact that its *generated formula* (4) is equivalent to (5):

$$((\varphi \rightarrow \varphi) \rightarrow \psi) \rightarrow \psi \tag{0.7}$$

which could be interpreted as stating that anything that follows from a formula like $\varphi \rightarrow \varphi$ could also be considered as such, given also that $\psi \rightarrow (\varphi \rightarrow \varphi)$ is usually a theorem of many logics.

In this paper, some conditions required to perform Carnap variations are explored, and their significance is also highlighted.

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Generalized set-assignment semantics for Parry systems

Nicolò Zamperlin

University of Cagliari, Italy

15:00–15:30

The proposal of my contribution is an algebraic-oriented approach to the study of content-sensitive connectives obeying a fully compositional behaviour, enterprise recently started by Thomas Ferguson with a Kripke-style approach (cf. [6], [7]). In my approach I extend the set-assignment semantics first developed by Richard Epstein ([5]).

Achieving a satisfying account of content-sensitivity in logic is a long-time goal. The first decisive step in this direction is Parry's logic of analytic implication ([10]), who expressed the relation that should hold between antecedent and consequent of an entailment in the form of the so-called proscriptive principle: "No formula with analytic implication as main relation holds universally if it has a free variable occurring in the consequent but not the antecedent." ([10], p.151). This principle is a pioneering attempt to implement a logical theory of subject-matter inclusion, which developed into various formalisms like relevance and containment logics. Among the most recent development in the field, there are Francesco Berto's works (e.g., [1], [2]). Berto introduces a modal language for a family of operators called topic-sensitive intensional modals (TSIMs). These are operators of the form $X^{\psi}\phi$, whose intuitive reading is "on the base of ψ, an agent Xs that ϕ", for some propositional attitude expressed by X. Berto further provides a possible world semantics for these operators, in which a necessary condition for the truth of $X^{\psi}\phi$ is that the content of ψ must be related to the content of ϕ. It is then shown how imposing conditions on such operators gives different reading of the modality, each suitable for modeling various hyperintensional phenomena. What this framework lacks, as pointed by Ferguson ([6]), is that in Berto's work the problem of the content of formulae involving any TSIM is left unanswered. In fact, while the formulae belonging to the non-modal fragment of the language are assigned a topic via a function t, that doesn't apply to formulae prefixed by a TSIM. Therefore the system is in this sense only first-degree, nested TSIMs are not allowed, restricting severely the expressive power of such languages.

In [6], Ferguson tries to provide a very weak system in which a content-sensitive intensional implication is both provided with an associated content without imposing any restriction on such assignment. His study starts from Parry's logic of analytic implication **PAI**. In the Kripke-style semantics provided by Fine in [8] for a language with an analytic implication $\to$, **PAI** is the logic charactized by a certain class of frames for modal logic **S4** in which every world is expanded with a join-semilattice. This provides the topics of each formula at that world, obtaining models of the form $\langle W, R, \langle T_w, \oplus_w\rangle_{w\in W}, v, t\rangle$, where $t : W \times Fm \to T_w$ is the topic function, that obeys $t_w(\neg\phi) = t_w(\phi)$ and $t_w(\phi \circ \psi) = t_w(\phi) \oplus t_w(\psi)$, for all binary $\circ$. In order to weaken the system to allow for the various interpretations of $\to$, Ferguson removes any condition on the content of implicative formulae, expanding the topic semilattice $\langle T_w, \oplus_w\rangle$ with a groupoid operation $\multimap$ s.t. $t_w(\phi \to \psi) = t_w(\phi) \multimap t_w(\psi)$. This implication is content-agnostic, as Ferguson calls it, since no condition are imposed over the respective operation on the topic semilattice. The resulting logic is **CA/PAI**, a subsystem of **PAI**.

Related to Parry's logic, there is its demodalization proposed by Dunn ([3]), which was independently rediscovered by Epstein ([4]) within his set-assignement semantics. This framework defines a model for a propositional language as a pair $\langle v, s\rangle$, where v is a two-valued valuation function and $s : Fm \to \mathcal{P}(X)$ maps each formula of the language to a subset of a fixed countable set X. The interaction between these two functions allows some connectives to become intensional. In the case of those which Epstein names relatedness and dependence logics, the only intesional connective is implication. A formula $\phi \to \psi$ has its standard Boolean value in case a specific relation holds between the intensional component of ϕ and ψ, namely $s(\phi)$ and $s(\psi)$, otherwise the implication is false. In the case of dependence logic **D** - equivalent to Dunn's **DAI** -, said relation is $s(\phi) \supseteq s(\psi)$. Set-assigment semantics is a versatile tool, in fact by tweaking the conditions on s we can obtain classes of models which characterize well known logics, like intuitionistic logic, various modal logics, many-valued logics like Łukasiewicz and strong Kleene logics. Despite its virtues, set-assignment semantics also has a serious limitation, which derives from the structure of the powerset $\mathcal{P}(X)$ that is the codomain of s. In order to generalize this approach, we want to be capable of choosing an arbitrary algebra as the codomain of s, in order to potentially get rid of some unwanted properties inherited from the powerset.

It is possible to generalize set-assignment semantics by allowing complete freedom in the choice of the algebra of truth-values (the 2-element Boolean algebra in Epstein's case) and the algebra of contents (that is $\mathcal{P}(X)$ for Epstein). Since these two algebras can have different types, we have to provide also a mean to translate the language in the type of the first one into the language of the second one. We can define a general Epstein model for a language of type ρ_0 as a tuple $\langle \mathbf{A}, \mathbf{B}, \nu, v_A, v_B \rangle$. $\mathbf{A}$ is an algebra of type $\rho_A \subseteq \rho_0$, $\mathbf{B}$ is an algebra of type ρ_B. $\nu : Fm_{\rho_0} \to Fm_{\rho_B}$ is a mapping that satisfies $\nu(x) = x$ and $\nu(\alpha(x_1/\beta_1, \dots x_n/\beta_n)) = \nu(\alpha)(x_1/\nu(\beta_1), \dots x_n/\nu(\beta_n))$ for all variables $x, x_1, \dots x_n \in Var$, and formulae $\alpha, \beta_1, \dots \beta_n \in Fm_{\rho_0}$. Finally $v_B : Fm_{\rho_B} \to B$ is a homomorphism, while $v_A : Fm_{\rho_0} \to A$ is a mapping which is a homomorphism only w.r.t. the symbols of ρ_A.

Considering a language of type $\langle \neg, \vee, \to \rangle$, where $\to$ is an intensional implication, Epstein's logic **D** can be easily recaptured via models of the form $\langle \mathbf{B}, \langle S, \oplus \rangle, \nu, v, s \rangle$, where $\mathbf{B}$ is any Boolean algebra, $\langle S, \oplus \rangle$ is a join-semilattice, ν behaves like the t function illustrated above for Fine's models for **PAI**, $s : \nu[Fm] \to S$ is a homomorphism, and v is a Boolean valuation with the exception that:

$$v(\phi \to \psi) = \begin{cases} v(\neg\phi \vee \psi) & \text{if } s(\nu(\psi)) \leq_\oplus s(\nu(\phi)) \\ 0^{\mathbf{B}} & \text{otherwise} \end{cases}$$

Moreover we can weaken the conditions on the translation mapping ν and obtain a content-agnostic system in the style of Ferguson's **CA/DAI**. We obtain models of the form $\langle \mathbf{B}, \langle S, \oplus, \multimap \rangle, \nu, v, s \rangle$, which differ from the previous ones for the fact that the join-semilattice $\langle S, \oplus \rangle$ is expanded with a groupoid operation $\multimap$: $S^2 \to S$, that is an operation with no further properties. Now $\nu(\phi \to \psi) = \nu(\phi) \multimap \nu(\psi)$. I axiomatized $\mathbf{DAI}_0$, the logic of this class of models, and proved it is equivalent to Ferguson's **CA/DAI**.

Returning to Parry's logic, no set-assignment semantics for **PAI** has been presented in the literature. What I provide is a first step in that direction, giving a generalized Epstein semantics w.r.t. which the *global* version of Parry's logic is complete. By global version, we intend Fine's axiomatization of **PAI** where the necessitation rule is substituted by its global counterpart: $(\text{Nec}_g)\ \phi \vdash \Box\phi$.

We start by Ferguson's subsystem **CA/PAI** and provide a semantics for its global version. For a modal language $\langle \neg, \vee, \Box, \to \rangle$, a PAI_0-model is a tuple $\langle \mathbf{B}^\Box, \langle S, \oplus, \multimap \rangle, \nu, v, s \rangle$, which differs from the models presented above for the logic $\mathbf{DAI}_0$ for the fact that the algebra of truth-values $\mathbf{B}^\Box$ is an interior algebra ([9]). The global modal logic induced by this class of models has been axiomatized and called $\mathbf{PAI}_0^g$. In order to obtain Parry's global logic, that is $\mathbf{PAI}^g$, we restrict the class of models to those s.t. their algebra of contents $\langle S, \oplus, \multimap \rangle$ satisfies $\oplus = \multimap$.

Completeness proofs have been provided for all systems, and equivalence proofs as well w.r.t. the logics **D** of Epstein and **CA/DAI** of Ferguson. While in the case of $\mathbf{PAI}^g$ and $\mathbf{PAI}_0^g$ these logics are the global versions of their respective counterparts **PAI** and **CA/PAI**, the usual relation between global and local modal logics still applies, that is the global logics are stronger than the local ones, while they coincide on their theorems.

The aim of the paper is to introduce generalized Epstein semantics and show its efficacy in the case study of Parry's systems and content-sensitive operators.

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Useful Information

The opening and the sessions on the 18th of June are held at the Aula (1st floor) of Collegium Novum (address: 24 Gołębia, Kraków).

The venue for the conference sessions for the rest of the days (19-21 June) will be at the JU Doctoral School in the Humanities, 34 Rynek Główny (Main Square), 2nd floor, 31-010 Kraków.