On the preservation of unification type of Heyting algebras and locally finite interior algebras.

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Previously, the results on unification types in intermediate logics (varieties of Heyting algebras) and their modal companions via Gödel-Tarski translation (varieties of interior algebras) were proved separately, although the final results were the same, see e.g. Ghilardi [4], [5], Dzik et al. [2]. Following McKinsey and Tarski [6], Blok [1] exhibited two functors between the category $\mathbb{IA}$ of interior algebras and the category $\mathbb{HA}$ of Heyting algebras

$$\mathcal{O}\colon \mathbb{IA} \to \mathbb{HA}, \quad \mathcal{B}\colon \mathbb{HA} \to \mathbb{IA}.$$

We will investigate the following questions:

1. If $\mathbf{V}$ is a variety of interior algebras and $A \in \mathbf{V}$, what is the unification type of A in $\mathbf{V}$ compared to the unification type of $\mathcal{O}(A)$ in $\mathcal{O}[\mathbf{V}]$? What is the type of $\mathbf{V}$ when compared to the type of $\mathcal{O}[\mathbf{V}]$?

2. If $\mathbf{V}$ is a variety of Heyting algebras and $L \in \mathbf{V}$, what is the unification type of L in $\mathbf{V}$ compared to the unification type of $\mathcal{B}(L)$ in $\mathbf{Eq}(\mathcal{B}[\mathbf{V}])$ [1]? What is the type of $\mathbf{V}$ when compared to the type of $\mathbf{Eq}(\mathcal{B}[\mathbf{V}])$?

We shall answer these questions if the varieties of interior algebras involved are locally finite Grzegorczyk algebras. We employ the algebraic approach to unification due to Ghilardi [3] which is based on finitely presented and projective objects.

[1] A variety generated by the free Boolean extensions of $L \in \mathbf{V}$

References

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