Categorical Duality for Algebraic Lattices

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Abstract

Algebraic lattices with complete lattice homomorphisms as morphisms form a category. On the other hand, join-semilattices with join preserving homomorphims $f\colon S \to S'$ which satisfy the following condition also form a category:

For each $a \in S$ and each nontrivial join cover $\{b, c\}$ of $f(a)$ in S', i.e. $f(a) \leq b \vee c$ and $f(a) \not\leq b$ and $f(a) \not\leq c$ there is a nontrivial join cover $\{d, e\}$ of a in S such that $\{f(d), f(e)\}$ refines $\{b, c\}$, i.e. $\{f(d), f(e)\}$ is below $\{b, c\}$ in terms of the partial order of S'.

The goal of our talk is to show that the two categories are dually equivalent in the Category Theory sense and show some benefits of this duality for Lattice Theory.

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