On some prospects of a connexive discussive logic

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A connexive logic is a theory with a connexive implication, that is, an implication for which:

(1) No proposition implies its own negation.
(2) No proposition implies each of two contradictory propositions.
(3) No proposition implies every proposition.
(4) No proposition is implied by every proposition.

(Pizzi and Williamson, 1997, p. 569)

Thus, in particular, following (3) no contradiction implies every proposition. So a paraconsistent logic is called for.

There are a few studies on connexive paraconsistent logics. On most prominent studies on this field, one can mention Omori (2016) where a connexive logic based on the paraconsistent logic **LP** has been developed. For the paraconsistent Brazilian tradition, some negative results have been proved by Ciuciura (2025). Ciuciura proved that there cannot be non-trivial connexive extensions of da Costa's paraconsistent systems C_n, $1 \leq n < \omega$. It is important to observe, however, that with the only exception of Estrada-González et al (unpublished), there have been almost no studies regarding discussive systems (another important tradition in paraconsistent logic) of connexive logics.

In this paper, we provide some logico-mathematical rudiments to develop discussive systems of connexive logics. We expand the modal connexive logic CK introduced by Wansing (2005). CK is characterized by the following axioms and rules:

(INT) all axioms schemes of intuitionistic positive logic.

(DNL)	$\sim\sim A \leftrightarrow A$	(K2)	$\Box A \land \Box B \to \Box(A \land B)$	(KT□)	$(\Diamond A \to \Box B) \to \Box(A \to B)$
(DM1)	$\sim(A \lor B) \leftrightarrow (\sim A \land \sim B)$	(L⊤)	$\Box(A \to A)$	(Df◊)	$\sim\Box A \leftrightarrow \Diamond\sim A$
(DM2)	$\sim(A \land B) \leftrightarrow (\sim A \lor \sim B)$	(M6)	$\Diamond(A \lor B) \to \Diamond A \lor \Diamond B$	(Df□)	$\sim\Diamond A \leftrightarrow \Box\sim A$
(HC)	$\sim(A \to B) \leftrightarrow (A \to \sim B)$	(K7)	$\Diamond(A \to B) \to (\Box A \to \Diamond B)$	(K9)	$\Box(A \lor B) \to \Box A \lor \Diamond$
(MP)	$A, A \to B/B$	$(\mathrm{RM}_\Box)$	$A \to B/\Box A \to \Box B$	$(\mathrm{RM}_\Diamond)$	$A \to B/\Diamond A \to \Diamond B$

We introduce CD, the connexive modal analogue of the deontic normal modal logic **D**. We obtain a completeness result for CD. Similarly as **D**, CD equals its $\Diamond$-counterpart. Based on CD we consider a discussive logic. We use some techniques employed in Mruczek-Nasieniewska and Nasieniewski (2019).

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