Suszko-Style RNmatrix Reductions: Truth-Functionality Lost

In this talk, we give a novel formalization of what we call *Suszko-style reductions*, using the machinery of *restricted non-deterministic matrices* (RNmatrices), as developed by Coniglio & Toledo in [3] and anticipated by, *e.g.*, Kearns [6] or Omori & Skurt [8]. Suszko-style reductions are generalizations of the original *Suszko's Reduction*, *i.e.*, the proof that every Tarskian consequence relation can be characterized by a two-valued semantics. Analogous (but many-valued) reductive results have since continued to develop for weaker, non-Tarskian logics.

Roman Suszko's result led to the controversial philosophical claim that, in effect, "every logic is [...] two-valued" ([9], p. 378), which has become known as *Suszko's Thesis* and has been subject to much criticism. One important point of criticism is that the resulting semantics is not always truth-functional. As Suszko says, "[e]ach adequate set of logical valuations bears a natural (dual) pre-topology which turns out to be a genuine topology in certain important cases." ([9], p. 378) In other important cases, however, it turns out not to be one.

More importantly, however, (non-truth-functional) *many-valued* semantics can be obtained using Suszko-style reductions for some non-Tarskian logics. This was first done by Grzegorz Malinowski in [7], who proved that every monotonic reflexive logic (q-consequence) has an at most three-valued semantics. An analogous result may be obtained for its dual, monotonic transitive logic (formalized by Szymon Frankowski as p-consequence in [4]). Moreover, recently, it has been proved in several settings that every monotonic logic has an at most four-valued semantics (by French & Ripley [5], Blasio, Marcos & Wansing [1], and Chemla & Égré [2]).

Following some remarks of Coniglio and Toledo's [3] as implicitly restating Suszko's Reduction in terms of their RNmatrix semantics, we propose that this result be extended to allow for a generalization in two directions. First, we generalize to the multiple-conclusion framework of *Scott* consequence relations, as opposed to the one-conclusion Tarskian logics. Second, we modify the RNmatrices to account for some weaker logics. The main formal results to be discussed are as follows:

Theorem 1. *Every Scott consequence relation is characterized by a two-valued RNmatrix.*

Theorem 2. *Every monotonic reflexive logic (p-consequence) is characterized by a three-valued RNmatrix.*

Theorem 3. *Every monotonic transitive logic (q-consequence) is characterized by a three-valued RNmatrix.*

Theorem 4. *Every purely monotonic logic is characterized by a four-valued RNmatrix.*

Our account of the Suszko-style phenomena has several notable advantages. As stated above, it not only generalizes to multiple conclusions but also provides a uniform treatment of all the reductions in tandem. Moreover, due to the use of non-deterministic matrices, it offers a natural explanation of the loss of truth-functionality obtained by the reductions.

The resulting RNmatrix semantics is straightforward and elegant, requires no extensive detours into model theory, and, more importantly, it is the only method known to us that offers a *matrix* semantics for the Suszko-style reductions. Philosophically, due to this observation, we propose that these results lead to some significant challenges for the use of RNmatrices as a perspicuous semantics.

References

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