ON SOME GENERALIZATION OF HEYTING LATTICES

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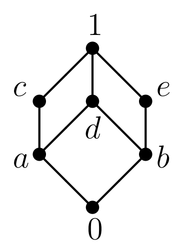
The class of Heyting lattices can be understood as the variety $(L, \vee, \wedge, \to, 1)$ satisfying the well-known axioms of distributive lattices with the unit, and in addition the identities:

$$x \wedge (x \to y) = x \wedge y, \quad x \wedge (y \to z) = x \wedge \big((x \wedge y) \to (x \wedge z)\big), \quad x \wedge \big((y \wedge z) \to y\big) = x.$$

The operation of relative pseudocomplementation

$$a \to b = \max\{x \in L : a \wedge x \leqslant b\}$$

does not, in general, exist in non-distributive lattices. For instance, in the following lattice



$c \to a$ is not properly defined.

In this talk, we present a broader class of lattices $(L, \vee, \wedge, \rightsquigarrow, 1)$, which satisfy the following conditions for all $a, b, x \in L$:

(a) $a \wedge (a \rightsquigarrow b) = a \wedge b$,
(b) $x \leqslant a \rightsquigarrow b \;\Rightarrow\; a \wedge x = a \wedge b$.

The operation $a \rightsquigarrow b = \max\{x \in L : a \wedge x = a \wedge b\}$ is well-defined in some non-distributive lattices (for instance, $c \rightsquigarrow a = d$ in the lattice shown above). We investigate the basic properties of $\rightsquigarrow$, compare it with the Heyting implication $\to$, and characterize the scope of this class. Finally, we turn to the question of the logic determined by this class of lattices.

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