A general approach in relational semantics for weak (modal) logics

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We present two types of relational structures which serve as semantic tools in studies on logics which require non-standard treatment of connectives or modal operators due to their hyperintensionality **[2, 7]**. We demonstrate how these structures, with appropriate definitions of truth, describe subnormal modal logics, including the system **N** known from **[3]** as well as its intuitionistic counterpart $\mathbf{N_{INT}}$ and their extensions which form classes of classical and intuitionistic intermediate – between **N** and **K** (the weakest classical normal modal logic), and between $\mathbf{N_{INT}}$ and $\mathbf{K_{INT}}$ – subnormal modal logics **[5]**. Furthermore, introduced structures provide semantics for a paraconsistent logic **CluN [1]**, thereby allowing for a translation between **CluN** and the subnormal modal logic axiomatised by $A \rightarrow \Box A$. Finally, the presented structures provide relational semantics for a 'failed axiomatisation' of **K** – a problem noted in **[4]** – and its extensions for which the only semantic theory known so far was non-deterministic semantics devised in **[6]**.

References

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