Some Unnoticed Paradoxes of Knowledge and Belief

Andrzej Wiśniewski

Department of Logic and Cognitive Science
Adam Mickiewicz University in Poznań, Poland
Andrzej.Wisniewski@amu.edu.pl

Three non-antinomic paradoxes concerning knowledge and belief will be discussed.

First, I will consider a generalized tripartite account of propositional knowledge. It will be represented by the following schema:

(TB^{+x}) $\quad \mathsf{K}p \equiv (p \wedge \mathsf{B}p) \wedge \Omega p$

where p is a propositional variable, K and B stand for the knowledge operator and the belief operator, respectively, and Ω is an unspecified expression by means of which the third condition of the tripartite account of knowledge is expressed. For decades or even centuries, philosophers discuss what the third condition should be. TB^{+x} is neutral in this controversy, however.

TB^{+x} leads to some, so far unnoticed, paradoxical consequences:

(PAC) $\quad \neg\mathsf{K}\neg p \wedge (\mathsf{B}\neg p \wedge \Omega\neg p) \rightarrow p$

(PDA) $\quad \mathsf{K}p \equiv \mathsf{B}p \wedge \mathsf{B}\Omega p$

The acronym PAC abbreviates 'Paradox of Astounding Consequent,' while PDA alludes to 'Paradox of Doxastic Agency.'

One can get PAC from the formula representing the generalized tripartite account, TB^{+x}, by CPL-means only: no specific assumptions concerning K, B, and Ω are needed. The antecedent of PAC is (proof-theoretically) consistent if only the underlying logic satisfies some natural conditions.

PDO, in contradistinction to PAC, is not just a CPL-consequence of TB^{+x}, but relies on some specific assumptions concerning knowledge, belief, and their interactions.

Second, I will consider the 'knowledge as true conviction' account of propositional knowledge. It will be represented by the following formula:

(TC) $\quad \mathsf{K}p \equiv p \wedge \mathsf{C}p$

where C stands for the conviction operator. It occurs that TC leads to the following paradoxical consequence:

(DMP) $\neg \mathsf{K}^{\mathsf{w}} p \to (\mathsf{C}\neg p \to p) \wedge (\mathsf{C}p \to \neg p)$

where K^{w} stands for knowledge-whether, defined in the standard way by the equivalence:

$$\mathsf{K}^{\mathsf{w}} p \equiv \mathsf{K}p \vee \mathsf{K}\neg p$$

The acronym DMP abbreviates 'Doxastic Misfortune Paradox.'

DMP does not rely on any specific assumptions concerning the conviction operator: one gets it by CPL-means if only knowledge is defined as true conviction and knowing-whether is conceived in the standard way.