EXPERIMENTS IN LOGIC AND THE ROLE OF DIAGRAMMATIC REPRESENTATIONS: A CASE STUDY OF ARISTOTELIAN SYLLOGISTIC PROOF CHARTS

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CONTENT

1. Introduction
2. Presentation of case study on Aristotelean syllogisms
3. Experiments
4. Free rides?
5. Conclusions

INTRODUCTION

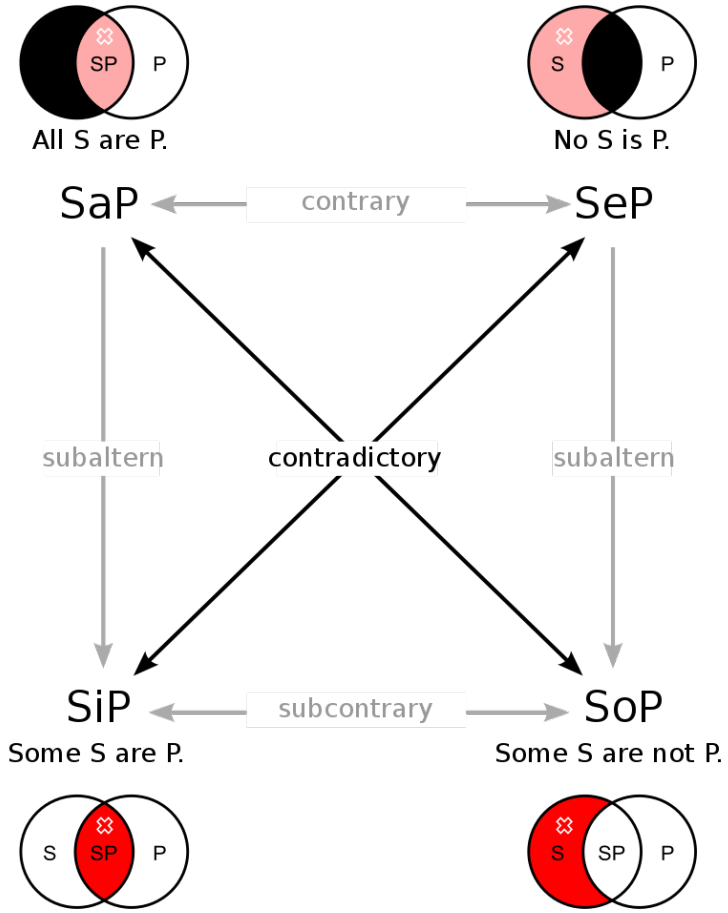
- The starting point from the practice of mathematics or logic
- Agent-based view
- Process of the knowledge production
  - The general theories (*The Growth of Mathematical Knowledge,* Breger & Grosholz 2000; *Proofs and Refutations*, Lakatos 1976)
  - The case study analysis (Carter 2010 and Starikova 2010) – how visual representations aid discoveries in analysis and algebra
  - Our example from logic - discoveries based on tables, not diagrams

INTRODUCTION

Aristotelian syllogistic

1. Universal affirmative statement: All S are P / SaP
2. Particular affirmative statement: Some S are P / SiP
3. Universal negative statement: No S are P / SeP
4. Particular negative statement: Some S are not P / SoP

INTRODUCTION



All M are P MaP
All S are M SaM
——— ———
All S are P SaP

- Barbara

No P are M PeM
All S are M SaM
——— ———
No S are P SeP

- Cesare

All M are P MaP
All S are M SaM
——— ———
Some S are P SiP

- Barbari

INTRODUCTION

Direct and indirect proofs in the interference relationships between the Moods from particular Figure (I, II, III, IV) – (the first Figure as the basis)

An example direct proof of Fresison (a fourth-Figure Mood) using Ferio (a first-Figure Mood):

1. PeM (major premise of Fresison)
2. MiS (minor premise of Fresison)
3. MeP (conversion applied on 1.)
4. SiM (conversion applied on 2.)
5. SoP (Ferio applied on 3. and 4., yelding the conclusion of Ferio)

INTRODUCTION

An example of indirect proof of Felapton (third-Figure Mood) using Datisi (another third-Figure Mood) looks like:

1. MeP (major premise of Felapton)
2. MaS (minor premise of Felapton)
3. SaP (law of contradiction applied on the conclusion of Felapton, SoP)
4. SiM (conversion applied on 2.)
5. MiP (Datisi applied on 3. and 4., yielding a contradiction to 1.)

DERIVING SYLLOGISMS

The process of finding the minimal set of inference rules needed to prove all syllogistic Moods (Wapniarski & Urbański, unpublished)

Four different main scenarios were first identified:

1. None of either conversions or subalternations are allowed to be used
2. Only the subalternations can be used
3. Only the conversions can be used
4. Both subalternations and conversions can be used

DESCRIPTION OF A CASE STUDY

Collecting the data

The series of four 25x25 tables („an imposed itself form”):

- four possible scenarios,
- the 24 valid Moods as columns (these ones used in the proofs),
- and 24 valid Moods as rows (the proved ones)
- existing proof marked by colored cells

DESCRIPTION OF A CASE STUDY

	Barbara	Celarent	Darii	Ferio	Barbari	Celaront	Cesare	Camestres	Festino	Baroco	Cesaro	Camestros	Darapti	Disamis	Datisi	Felapton	Bocardo	Ferison	Bramantip	Camenes	Dimaris	Fesapo	Fresison	Camenos
Barbara										■							■							
Celarent			■	■					■					■	■			■			■		■	
Darii		■		■			■	■	■									■		■			■	
Ferio		■	■				■	■						■	■					■	■			
Barbari		■		■			■	■	■			■				■		■		■		■	■	■
Celaront		■	■	■		■	■	■	■		■		■	■	■			■		■	■		■	
Cesare			■	■					■					■	■			■			■		■	
Camestres			■	■					■					■	■			■			■		■	
Festino		■	■				■	■						■	■					■	■			
Baroco	■																■							
Cesaro		■	■	■		■	■	■	■		■		■	■	■			■		■	■		■	
Camestros	■		■	■	■				■					■	■	■		■	■		■	■	■	
Darapti		■		■		■	■	■	■		■							■		■			■	
Disamis		■		■			■	■	■									■		■			■	
Datisi		■		■			■	■	■									■		■			■	
Felapton	■	■	■		■		■	■				■		■	■				■	■	■			■
Bocardo	■									■														
Ferison		■	■				■	■						■	■					■	■			
Bramantip		■		■			■	■	■			■				■		■		■		■	■	■
Camenes			■	■					■					■	■			■			■		■	
Dimaris		■		■			■	■	■									■		■			■	
Fesapo	■	■	■		■		■	■				■		■	■				■	■	■			■
Fresison		■	■				■	■						■	■					■	■			
Camenos	■		■	■	■				■					■	■	■		■	■		■	■	■	

Table 1: Conversion-only scenario

DESCRIPTION OF A CASE STUDY

Playing with the tables

- Certain columns (for Moods Cesare and Camestres) look alike,
- Certain column (for Festino) is somewhat regular,
- Certain columns (Baroco, Cesaro, Camestro) have much less proof-cases indicated.

Rearranging the table:

This step turned out to be crucial, but was not planned;

the idea of rearranging the tables came from noticing visual regularities on the tables.

DESCRIPTION OF A CASE STUDY

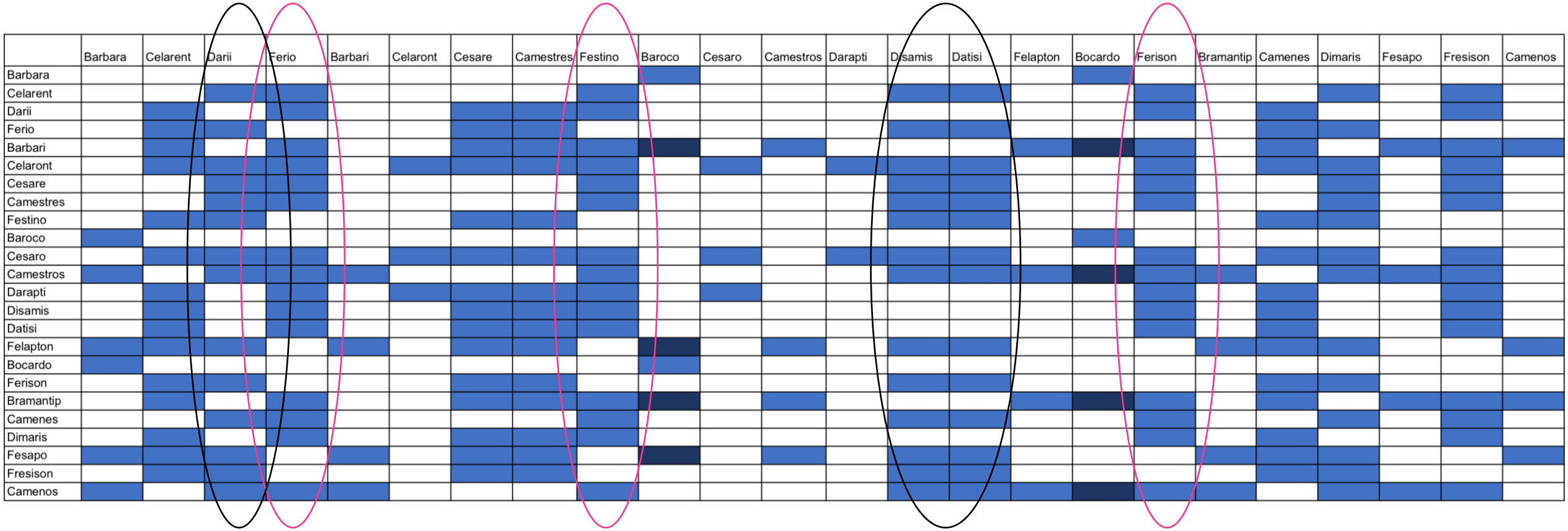
	Barbara	Baroco	Bocardo	Celarent	Cesare	Camestres	Camenes	Darii	Disamis	Datisi	Dimaris	Ferio	Festino	Ferison	Fresison	Bramantip	Barbari	Fesapo	Felapton	Celaront	Cesaro	Camestros	Camenos	Darapti
Barbara		■	■																					
Baroco	■		■																					
Bocardo	■	■																						
Celarent								■	■	■	■	■	■	■	■									
Cesare								■	■	■	■	■	■	■	■									
Camestres								■	■	■	■	■	■	■	■									
Camenes								■	■	■	■	■	■	■	■									
Darii				■	■	■	■					■	■	■	■									
Disamis				■	■	■	■					■	■	■	■									
Datisi				■	■	■	■					■	■	■	■									
Dimaris				■	■	■	■					■	■	■	■									
Ferio				■	■	■	■	■	■	■	■													
Festino				■	■	■	■	■	■	■	■													
Ferison				■	■	■	■	■	■	■	■													
Fresison				■	■	■	■	■	■	■	■													
Bramantip		■	■	■	■	■	■					■	■	■	■			■	■			■	■	
Barbari		■	■	■	■	■	■					■	■	■	■			■	■			■	■	
Fesapo	■	■		■	■	■	■	■	■	■	■					■	■					■	■	
Felapton	■	■		■	■	■	■	■	■	■	■					■	■					■	■	
Celaront				■	■	■	■	■	■	■	■	■	■	■	■					■	■			■
Cesaro				■	■	■	■	■	■	■	■	■	■	■	■					■	■			■
Camestros	■		■					■	■	■	■	■	■	■	■	■	■	■	■					
Camenos	■		■					■	■	■	■	■	■	■	■	■	■	■	■					
Darapti				■	■	■	■					■	■	■	■					■	■			

Table 2: Conversions plus subalternations after rearrangement

DESCRIPTION OF A CASE STUDY

- The examples (subalternations plus conversions) with only the indirect proofs marked, and with the indirect- and direct-proofs as well
- First table: One can read off that certain syllogisms clearly form distinct groups exhibiting different behaviors
- Second table: more regularities

DESCRIPTION OF A CASE STUDY



	Barbara	Celarent	Darii	Ferio	Barbari	Celaront	Cesare	Camestres	Festino	Baroco	Cesaro	Camestros	Darapti	Disamis	Datisi	Felapton	Bocardo	Ferison	Bramantip	Camenes	Dimaris	Fesapo	Fresison	Camenos
Barbara										■							■							
Celarent			■	■					■					■	■			■			■		■	
Darii		■		■			■	■	■									■		■			■	
Ferio		■	■				■	■						■	■					■	■			
Barbari		■		■			■	■	■	■		■				■	■	■		■		■	■	■
Celaront		■	■	■		■	■	■	■		■		■	■	■			■		■	■		■	
Cesare			■	■					■					■	■			■			■		■	
Camestres			■	■					■					■	■			■			■		■	
Festino		■	■				■	■						■	■					■	■			
Baroco	■																■							
Cesaro		■	■	■		■	■	■	■		■		■	■	■			■		■	■		■	
Camestros	■		■	■	■				■					■	■	■	■	■	■		■		■	
Darapti		■		■		■	■	■	■		■							■		■			■	
Disamis		■		■			■	■	■									■		■			■	
Datisi		■		■			■	■	■									■		■			■	
Felapton	■	■	■		■		■	■		■		■		■	■				■	■	■			■
Bocardo	■									■														
Ferison		■	■				■	■						■	■					■	■			
Bramantip		■		■			■	■	■	■		■				■	■	■		■		■	■	■
Camenes			■	■					■					■	■			■			■		■	
Dimaris		■		■			■	■	■									■		■			■	
Fesapo	■	■	■		■		■	■		■		■		■	■				■	■	■			■
Fresison		■	■				■	■						■	■					■	■			
Camenos	■		■	■	■				■					■	■	■	■	■	■		■	■	■	

Table 3: First Figure in subalternations and conversions scenario before adding direct proofs

DESCRIPTION OF A CASE STUDY

	Barbara	Celarent	Darii	Ferio	Barbari	Celaront	Cesare	Camestres	Festino	Baroco	Cesaro	Camestros	Darapti	Disamis	Datisi	Felapton	Bocardo	Ferison	Bramantip	Camenes	Dimaris	Fesapo	Fresison	Camenos
Barbara	■									■							■							
Celarent		■	■	■			■	■	■					■	■			■		■	■		■	
Darii		■	■	■			■	■	■					■	■			■		■	■		■	
Ferio		■	■	■			■	■	■					■	■			■		■	■		■	
Barbari	■	■	■	■	■		■	■	■	■		■		■	■	■	■	■	■	■	■	■	■	■
Celaront		■	■	■		■	■	■	■		■		■	■	■			■		■	■		■	
Cesare		■	■	■			■	■	■					■	■			■		■	■		■	
Camestres		■	■	■			■	■	■					■	■			■		■	■		■	
Festino		■	■	■			■	■	■					■	■			■		■	■		■	
Baroco	■									■							■							
Cesaro		■	■	■		■	■	■	■		■		■	■	■			■		■	■		■	
Camestros	■	■	■	■	■		■	■	■	■		■		■	■	■	■	■	■	■	■	■	■	■
Darapti		■	■	■		■	■	■	■		■		■	■	■			■		■	■		■	
Disamis		■	■	■			■	■	■					■	■			■		■	■		■	
Datisi		■	■	■			■	■	■					■	■			■		■	■		■	
Felapton	■	■	■	■	■		■	■	■	■		■		■	■	■	■	■	■	■	■	■	■	■
Bocardo	■									■							■							
Ferison		■	■	■			■	■	■					■	■			■		■	■		■	
Bramantip	■	■	■	■	■		■	■	■	■		■		■	■	■	■	■	■	■	■	■	■	■
Camenes		■	■	■			■	■	■					■	■			■		■	■		■	
Dimaris		■	■	■			■	■	■					■	■			■		■	■		■	
Fesapo	■	■	■	■	■		■	■	■	■		■		■	■	■	■	■	■	■	■	■	■	■
Fresison		■	■	■			■	■	■					■	■			■		■	■		■	
Camenos	■	■	■	■	■		■	■	■	■		■		■	■	■	■	■	■	■	■	■	■	■

Table 4: Subalternations and conversions scenario after adding direct proofs

DESCRIPTION OF A CASE STUDY

Many Moods can be treated as forming distinct groups, and that these groups can be graphically represented by one of their constituent members.

	Barbara-Baroco-Bocardo	C, D, and F group	Subalternated group	Celaront-Cesaro-Darapti
Barbara-Baroco-Bocardo				
C, D, and F group				
Subalternated group				
Celaront-Cesaro-Darapti				

Table 5: Final table with reduced groups (conversions and subalternations)

DESCRIPTION OF A CASE STUDY

Additional function of visual (spatial) representations:

- Free rides: syllogistic moods in this group can be proved mutually only by themselves:

	Barbara	Celarent	Darii	Ferio	Barbari	Celaront	Cesare	Camestres	Festino	Baroco	Cesaro	Camestros	Darapti	Disamis	Datisi	Felapton	Bocardo	Ferison	Bramantip	Camenes	Dimaris	Fesapo	Fresison	Camenos
Barbara										■							■							
Celarent									■					■										
Darii								■										■						
Ferio							■								■									
Barbari								■		■		■				■	■	■						
Celaront							■		■		■		■	■	■									
Cesare				■											■									
Camestres			■															■						
Festino		■												■										
Baroco	■																■							
Cesaro		■		■		■							■	■	■									
Camestros	■		■		■											■	■	■						
Darapti		■		■		■	■		■		■													
Disamis		■							■															
Datisi				■			■																	
Felapton	■		■		■			■		■		■												
Bocardo	■									■														
Ferison			■					■																
Bramantip																				■		■	■	■
Camenes																					■		■	
Dimaris																				■			■	
Fesapo																			■	■	■			■
Fresison																				■	■			
Camenos																			■		■	■	■	

Table 6: The distinctness of the fourth-Figure in first scenario (subalternation)

- Self-correcting (navigational) role

EXPERIMENTS IN FORMAL SCIENCES

Considered definition of experiment:

„*the setup, which includes the research question, a process with an outcome, and the conclusion*"

(Schlimm & Gonzalez 2024)

EXPERIMENTS IN FORMAL SCIENCES

Experimental steps:

1. Initial **research question**: how to find the minimal set of inference rules needed to prove all syllogistic Moods
2. **A proces with an outcome**:
   "calculating" by hand/Isabelle the dependence between all the syllogistic Moods,
3. **Additional element (question and the proces with outcome) of experiment**:
   Consideration on how to present the outcomes in a comprehensible way (constructing and playing with a two-dimensional table)
4. **Conclusions**: unexpected information, forms of presentations of a data (of beginning outcomes) affect what can be read off.

FREE RIDES

The tables and diagrams

- Different ways to show relations (Carter 2024; Bertin 2010) - diagram represents relations by lines or arrows, tables display, or instantiates relations (but there are colored cells)
- diagrams need not have a unique interpretation (Grosholz, 2007; Macbeth 2012)
- diagrams and three levels: 1. the visual appearance of the representation; 2. the semantics and the interpretations of the sign; 3. the using diagrams in contrast to linguistic representations (Carter 2021)

FREE RIDES

Free rides:

Giaquinto (2007), Carter (2021), Shimojima (1996, 1999)

Carter's definition: "*the phenomenon that a new, consequential piece of information can be obtained from an external representation, information that has not been added when constructing it*" (Carter 2021, p. 10476).

FREE RIDES

Different types of information obtained from **visual representations** (Carter 2021, p. 10478):

„1. A representation gives rise to new properties of concepts.
2. A representation shows relations between properties.
3. A representation gives rise to new concepts which can be used to reformulate (and visualise) a given condition.
4. A representation is used as a navigation tool.
5. A representation is used as an organisatorial devise, to systematise.
6. A representation gives rise to a new (and easier) calculation."

FREE RIDES

Additional types of information obtained from **visual representations (our tables):**

7. A representation is used as an auto-checking tool, to correct errors in the organised, linguistic data
8. A representation is used to manipulate the iconistic (or partially iconistic) data, and in this way starts to show relations between properties which would otherwise be hard to spot.

FREE RIDES

Internal representations (mental, cognition schemes)

It is assumed that the visual system stores representations of configuration types. Where the representation is not an image, but:

- „(i) a visual stimulus can activate a representation of a type (…)
- (ii) we can activate a type representation to produce an image of an instance of the type by voluntary visual imagining." (Giaquinto 2008, 46)

The visual category specification provides a more flexible and direct way of grasping the structure than the template (Giaquinto 2008)

- Explanation for the context of Giaquinto's tools (M. Friedman, C.J. Rittberg 2019)

FREE RIDES

Internal representations (mental, cognitive schemes)

- the stage of creating tables
  - the knowledge of Aristotelian syllogistic, the form of the table was an imposing one (the spatial orientation system
  - the up-down, right-left relations
  - the relations of entailment between the individual elements of the "set of modes"
- certain regularities were noticed at the visual level – observing similar or different "behavior" (patterns) for entire groups of syllogisms and modes
- observing and correcting some erroneous proofs
- the observation that the fourth figure preserves a certain distinctiveness - this information "implied itself"

CONCLUSIONS

Free rides in the analysed logical experiment case:

- the information read off as “groups look alike”, when the information introduced was the logical dependence between particular Moods (by shaded cell or nor)
- the observation that the fourth figure preserves a certain distinctiveness

Outcomes of experiments depend on how information is presented.

Using diverse mental representations increases the efficiency of the experiment (Tytko et al., 2023).

. Where efficiency is understood as:

- the shortening of the experiment time,
- the appearance of unentered information,
- the improvement of correctness of the proces

BIBLIOGRAPHY

- Bertin, J. (2010). Semiology of Graphics (W. J. Berg, Trans.). Esri Press.
- Carter, J. (2010). Diagrams and Proofs in Analysis. International Studies in the Philosophy of Science, 24 (1), 1–14. https : / / doi . org / 10 . 1080 / 02698590903467085
- Carter, J. (2018). Graph-algebras—Faithful representations and mediating objects in mathematics. Endeavour, 42 (2), 180–188. https : / / doi . org / https://doi.org/10.1016/j.endeavour.2018.07.006
- Carter, J. (2021). "Free rides" in Mathematics. Synthese, 199, 10475–10498. https://doi.org/10.1007/s11229-021-03255-9
- De Toffoli, S. (2022). What Are Mathematical Diagrams? Synthese, 200 (2), 1– 29. https://doi.org/10.1007/s11229-022-03553-w
- De Toffoli, S. (2023). Who's Afraid of Mathematical Diagrams? Philosophers' Imprint, 23 (1). https://doi.org/10.3998/phimp.1348
- Giaquinto, M. (2007, May). Visual Thinking in Mathematics: An Epistemological Study. Oxford University Press. https://doi.org/10.1093/acprof:
- oso/9780199285945.001.0001
- Grosholz, E. R. (2007). Representation and Productive Ambiguity in Mathematics and the Sciences. Oxford University Press.
- Grosholz, E. R., & Breger, H. (Eds.). (2000). The Growth of Mathematical Knowledge. Kluwer Academic Publishers.
- Koutsoukou-Argyraki, A., & Wapniarski, K. (2025). A Formalisation of Aristotle's Assertoric Syllogistic in Isabelle/HOL. Topoi – An International Review of Philosophy, (Special Issue: Meaning and Understanding via Proofs). https://doi.org/10.1007/s11245-025-10184-6
- Lakatos, I. (1976). Proofs and Refutations: The Logic of Mathematical Discovery (J. Worrall & E. Zahar, Eds.). Cambridge University Press.

BIBLIOGRAPHY

- Leibniz, G. W. (1966). Logical Papers: A Selection Translated and Edited with an Introduction. Oxford University Press.
- Parsons, T. (2014). Articulating Medieval Logic. Oxford University Press.
- Parsons, T. (2021). The Traditional Square of Opposition. In E. N. Zalta (Ed.), The Stanford encyclopedia of philosophy (Fall 2021). Metaphysics Research Lab, Stanford University.
- Rose, L. E. (1965). Aristotle's Syllogistic and the Fourth Figure. Mind. New Series, 74 (295), 382–89.
- Schlimm, D., & Gonz´alez, J. F. (2024). Mathematical experiments on paper and computer. In B. Sriraman (Ed.), Handbook of the history and philosophy of mathematical practice. Springer, Cham. https://doi.org/10.1007/978-3-031-40846-5 112
- Shimojima, A. (1996). On the Efficacy of Representation [PhD thesis]. Indiana University.
- Shimojima, A. (1999). The Graphic-Linguistic Distinction: Exploring Alternatives. Artificial Intelligence Review, 13, 313–335. https://doi.org/10.1023/A:1006544013197
- Smith, R. (2022). Aristotle's Logic. In E. N. Zalta & U. Nodelman (Eds.), The Stanford encyclopedia of philosophy (Winter 2022). Metaphysics Research Lab, Stanford University.
- Starikova, I. (2010). Why Do Mathematicians Need Different Ways of Presenting Mathematical Objects? The Case of Cayley Graphs. Topoi – An International Review of Philosophy, 29 (1), 41–51. https://doi.org/10.1007/s11245-009-9065-4
- Tytko, Karolina, Franci Mangraviti, Natalia Rylko, Nurtazina Karlygash B. 2023. „New epistemology of mathematics and formal sciences in the age of AI. Critical concept kinds and diversity of mental representations". *Edukacja Biologiczna i Środowiskowa* 2 (80). https://doi.org/10.24131/3247.230207.
- Wapniarski, K., & Urba´nski, M. (unpublished). Aristotelian Syllogistic more apagogico demonstrata: Identifying Minimal Sets of Inference Rules for Indirectly Proving All the Syllogistic Moods [manuscript sent for publication].

THANK YOU FOR THE ATTENTION ☺