NON-REDUCIBLE INFINITE THEORIES OF THE FIRST ORDER

ADAM MATA
Faculty of Mathematics and Information Science, Warsaw University of Technology
adam.mata.dokt@pw.edu.pl

The following fact is a folklore in model theory:

Theorem 1 *Let T be an infinite, consistent first-order theory. If*

- *T has no finite model,*
- *every finite subset of T has a finite model*

then T is not equivalent to any finite theory.

We provide a proof analysis of this fact which allows us to obtain a more general theorem:

Theorem 2 *Let κ be some infinite cardinal. Let T be a consistent first order theory of cardinality κ. If every proper infinite subset $T' \subseteq T$, such that $|T'| < |T|$ has a model $M_{T'}$ which at the same time is not a model of T then T is not equivalent with any other theory $\widehat{T}$ such that $|\widehat{T}| < \kappa$.*

We also provide examples of some established theories which cannot be simplified as well as remarks on the generalized theorem.

[1] R. Cori, D. Lascar, *Mathematical Logic Part II*, Oxford University Press, 2001.