

Lax amalgamation and equational consequence

Wesley Fussner

The Czech Academy of Sciences, Institute of Computer Science
fussner@cs.cas.cz

A class K of mathematical structures has the amalgamation property when any two structures $\mathbf{B}, \mathbf{C} \in K$ with a common substructure $\mathbf{A}$ may be embedded in some $\mathbf{D} \in K$ so that the embeddings coincide on the common substructure $\mathbf{A}$. The ramifications of the amalgamation property for high-level features of the equational consequence relation of K are well known [2], and form the core motivation for investigating amalgamation when K is an algebraic semantics for some logic.

In this talk, we discuss several natural generalizations of this set-up. In particular, we drop the stipulation that $\mathbf{B}$ and $\mathbf{C}$ embed into $\mathbf{D}$ and require only that there are homomorphisms from $\mathbf{B}$ and $\mathbf{C}$ into $\mathbf{D}$, arriving at a lax variant of amalgamation previously appearing in, for example, [3]. We discuss the latter in terms of equational consequence and consider several case studies from algebraic logic and general algebra.

This talk is based on joint work with Josef von Hoffmann Doyle and Krzysztof Krawczyk.

References

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- [3] L.L. Maksimova. Restricted interpolation in modal logics. In *Advances in Modal Logic*, Vol. 4 (2003), pp. 297–311.