Decision problems for ordered monoids

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This talk will be about deciding equations in various classes of ordered monoids and how this relates to properties of spaces of (pre)orders on certain monoids. For example, the equational theory of lattice-ordered abelian groups is decidable and corresponds to deciding if there exists an order satisfying a finite set of inequalities on a free abelian group. Similarly, we will see that the equational theory of distributive lattice-ordered monoids is decidable and corresponds to deciding if there exists a right order satisfying a finite set of inequalities on a free group [1, 3]. Finally, we will see how descriptions of spaces of preorders on commutative monoids can be used to establish the decidability of the universal theory of ordered commutative monoids [2].

References

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[2] J. Draisma, G. Metcalfe, and S. Santschi. The space of preorders on a commutative monoid. Manuscript, 2026.

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