Machine Space, Exponentials and Compactness

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3) If P and Q are verifiable, then $P \wedge Q$ is verifiable $\longrightarrow$ If $U, V \in \mathcal{T}$ then $U \cap V \in \mathcal{T}$
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An explanation for this will follow from the solution to another problem.

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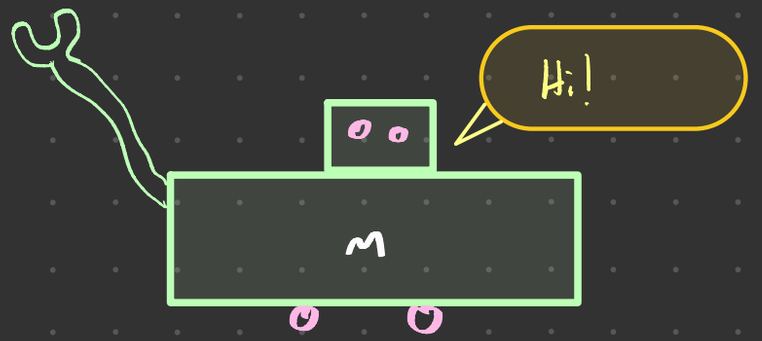

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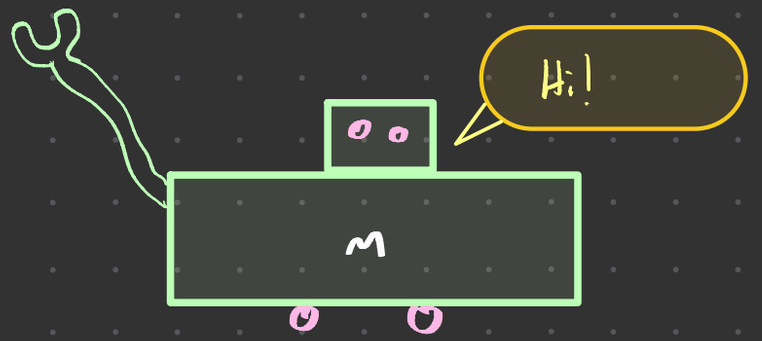

The idea that will resolve both of the specified problems will be to introduce a distinction between verifiable properties and procedures which carry out these verification procedures.

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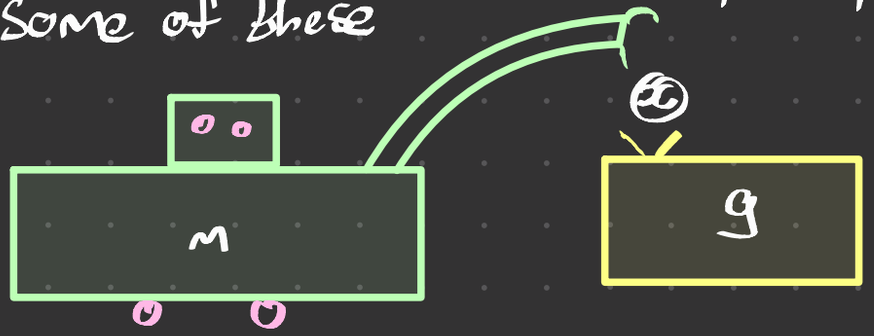

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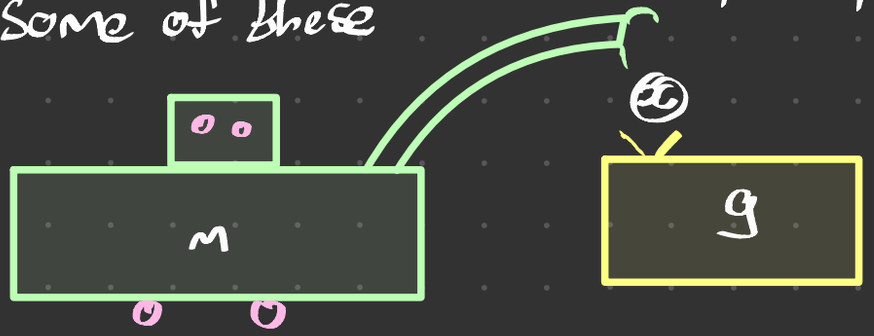

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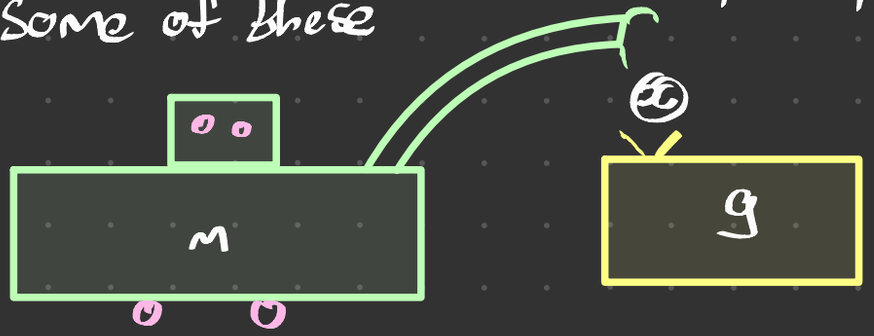

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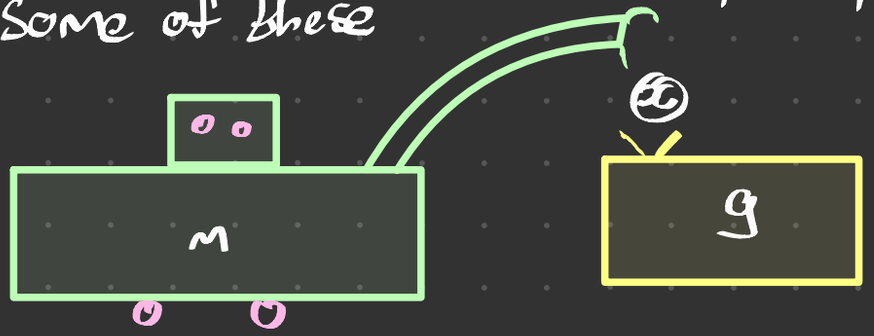

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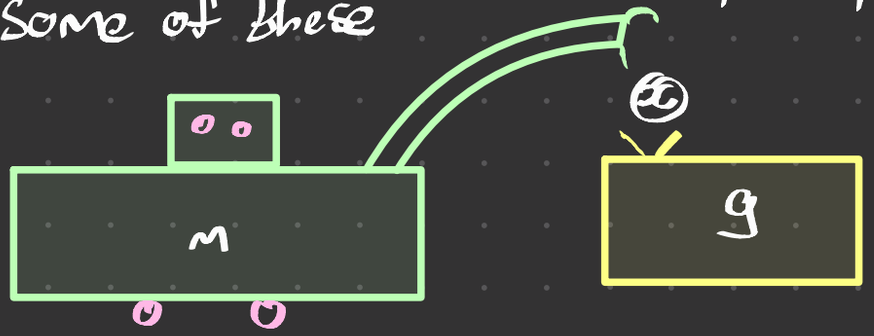

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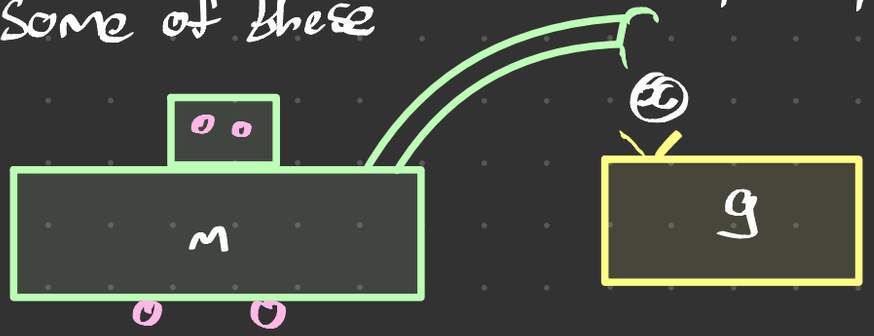

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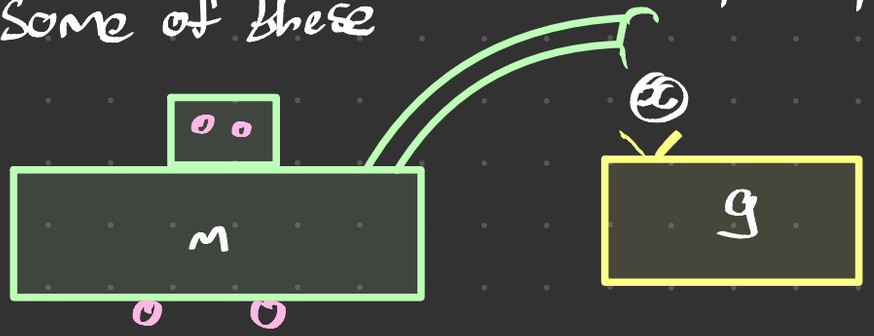

After placing a point into some predefined black boxes, the machine either indicates success after some finite time, or it waits forever.

The robot progresses through different stages.

1) On the i'th stage it selects finitely many black boxes.
2) It writes the number i on them
3) Runs each box with input x.
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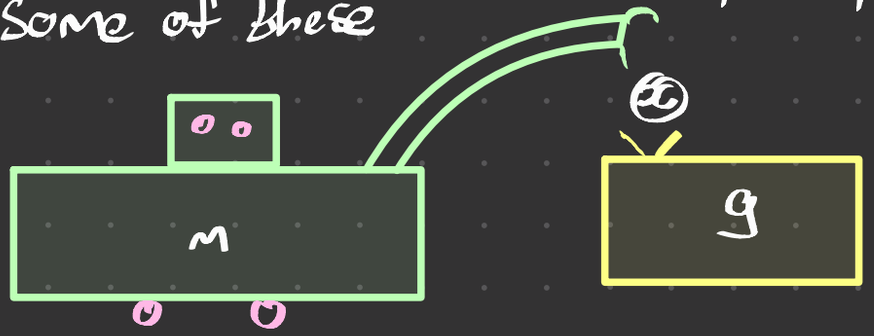

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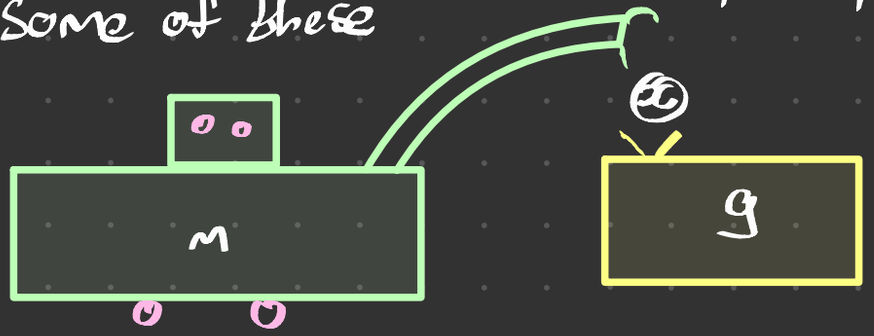

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These robots are the opens of Σ^G

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runs f...

We ...
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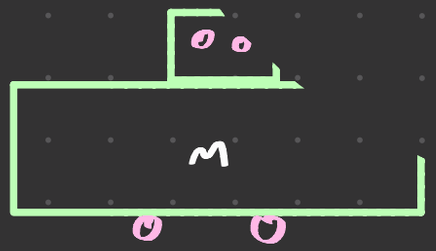

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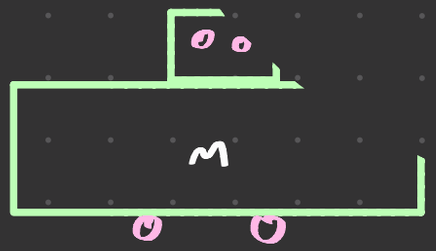

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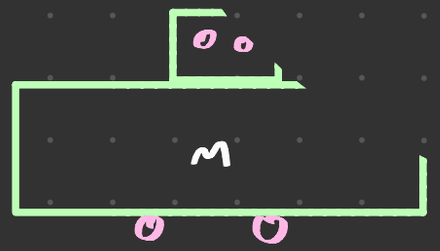

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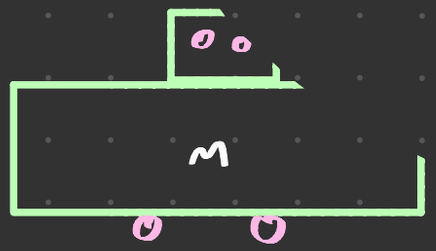

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where a machine $m = \bigvee_{i \in I} \bigwedge_{j \in J_i} g_{ij}$ lies in $\boxtimes F$ if and only if there exists J_k such that

$$\forall_{j \in J_k} \; g_{kj} \in F$$

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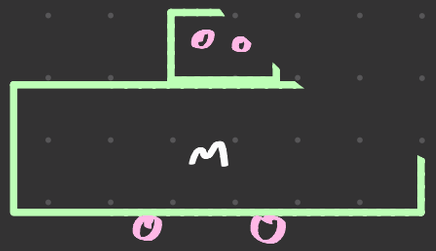

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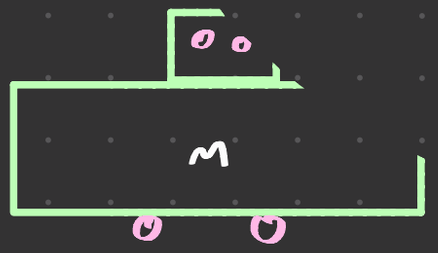

Simply watch the machine operate to verify this!

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Weak Exponentials

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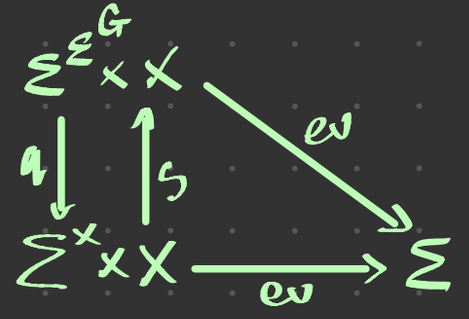

This diagram commutes and tells us the way to determine if x lies in an open is to find a machine that corresponds to it, and apply it to x.

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Note that this precisely resolves the issue of exponentials.

The exponential exists (i.e the space is locally compact) precisely when there exist a continuous assignment of opens to machines

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Verifiable Universal Quantification

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Crucially we do n
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To see this in action check out "seemingly impossible functional programs"

By Martin Escardo