Exactly Five Subquasivarieties of Sugihara have the Amalgamation Property

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Amalgamation in varieties of algebras

Following [Maksimova, 1977] celebrated result – research on AP has been restricted to lattices of subvarieties of algebras,
[Di Nola and Lettieri, 2000], [Metcalfe et al., 2014],
[Fussner and Santschi, 2025b] [Fussner and Santschi, 2024],
[Fussner and Santschi, 2025a], [Marchioni and Metcalfe, 2012]

This conference: (Almeida, Chen); (Bezhanishvili, Chen); (Doyle, Fussner); (Doyle, Fussner, K).

the quasivariety challenge

Results on amalgamation predominantly restricted to varieties.
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Let Q *be any quasivariety with the* RCEP *such that* $\mathsf{Q}_{\mathrm{RFSI}}$ *is closed under subalgebras. Then* Q *has the* AP *if and only if every span of algebras in* $\mathsf{Q}_{\mathrm{FG}} \cap \mathsf{Q}_{\mathrm{RFSI}}$ *has an amalgam in* Q.

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Hard to apply in quasivarieties because of two hard problems:

(1) We usually have a bad handle on R(F)SI's in quasivarieties (lack of good characterizations)
(2) RCEP usually fails in (sub)quasivarieties even if the initial variety has it

The general goal

Extend the understanding of the AP phenomenon beyond varieties.

Obtain a Maksimova style characterization for a lattice of subquasivarieties.

We do it for Sugihara algebras because:

- Sugiharas behave "well";
- they correspond to the well-known and historically important system of relevance logic RM;
- we want to draw "logical" corollaries about interpolation-like properties.

Amalgamation Property

A class K has AP iff every span $\langle \varphi_B : A \hookrightarrow B, \varphi_C : A \hookrightarrow C \rangle$ in K can be completed in K (has an amalgam in K) i.e. there exists $D \in K$ and embeddings $\langle \psi_B : B \hookrightarrow D, \psi_C : C \hookrightarrow D \rangle$ such that $\psi_B \varphi_B = \psi_C \varphi_C$.

$$\begin{tikzcd} & B \\ A \arrow[ur, hook, "\varphi_B"] \arrow[dr, hook, "\varphi_C"'] & \\ & C \end{tikzcd}$$

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 & C \arrow[ur, hook, dashed, "\psi_C"'] &
\end{tikzcd}$$

Transferable injections TIP – just diagramatically

$$
\begin{tikzcd}
 & B \arrow[dr, hook, dashed, red, "\psi_B"] & \\
A \arrow[ur, "\varphi_B"] \arrow[dr, hook, "\varphi_C"'] & & \color{red}{D} \\
 & C \arrow[ur, dashed, red, "\psi_C"'] &
\end{tikzcd}
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Relative congruence extension property

Given a class of algebras K, and an algebra $\mathfrak{A}$, a congruence $\Theta \in Con(\mathfrak{A})$ is said to be a K–congruence when $\mathfrak{A}/\Theta \in \mathsf{K}$. The set of K-congruences of $\mathfrak{A}$ will be denoted by $Con_{\mathsf{K}}(\mathfrak{A})$.

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A known algebraic fact:
TIP $\Leftrightarrow$ AP + RCEP

Sugihara algebras

$\mathrm{Z} = \langle \mathrm{Z}, \wedge, \vee, \rightarrow, \neg \rangle$,

$$x \rightarrow y = \begin{cases} (-x) \vee y & x \leq y \\ (-x) \wedge y & x \not\leq y. \end{cases}$$

Sugihara algebras are memebers of $\mathbb{V}(\mathrm{Z}) = HSP(\mathrm{Z})$.

Subchains (finitely subdirectly irreducibles)

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Further, for each integer $n \geq 0$, each of the set $\{-2n-1, -2n, \ldots, -1, 0, 1, \ldots, 2n, 2n+1\}$ gives the universe of an algebra of Z that we denote Z_{2n+1}, and,

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for $n \geq 1$, each of the sets $\{-2n-1, -2n, \ldots, -1, 1, \ldots, 2n, 2n+1\}$ gives the universe of a subalgebra Z_{2n} of E.

Subvarieties of Sugihara algebras

It turns out that the lattice of subvarieties of SA forms a countable chain given by

$$\mathbb{V}(\mathsf{Z}_1) \subseteq \mathbb{V}(\mathsf{Z}_2) \subseteq \mathbb{V}(\mathsf{Z}_3) \subseteq \cdots \mathbb{V}(\mathsf{Z}) = \mathbb{V}(\mathsf{E}) = \mathsf{SA}.$$

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Further, $\mathsf{V}(\mathsf{Z}) = \mathbb{Q}(\mathsf{Z}) = \mathbb{Q}(\{\mathsf{Z}_n \mid n \geq 1\}) = \mathbb{Q}(\{\mathsf{Z}_{2n+1} \mid n \geq 0\})$, and also $\mathbb{Q}(\mathsf{E}) = \mathbb{Q}(\{\mathsf{Z}_{2n} \mid n \geq 1\})$ is a proper subquasivariety of SA.

Our result

Theorem (Algebra)

There are exactly five subquasivarieties of Sugihara algebras with Amalgamation Property.

Important results/useful lemmas

Lemma ([Czelakowski and Dzobiak, 1999])

All subquasiivarieties of Sugihara algebras with RCEP are the following: $\mathbb{V}(\mathrm{Z}) \cup \{\mathbb{V}(\mathrm{Z}_n) : n \in \omega\} \cup \{\mathbb{Q}(\mathrm{E})\} \cup \{\mathbb{Q}(\mathrm{Z}_{2n}) : n \in \omega\}$.

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Lemma ([Krawczyk, 2022])

Let Q be a quasivariety such that $\mathbb{Q}(\mathrm{Z}_2) \subsetneq Q$. Then either $\mathrm{Z}_2 \times \mathrm{Z}_3 \in Q$ or $\mathrm{Z}_2 \times \mathrm{Z}_4 \in Q$.

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Lemma ([Gil-Férez et al., 2020])

The class of totally ordered odd Sugihara monoids has the amalgamation property.

Positive part

Lemma

Each of the quasivarieties $\mathbb{V}(\mathrm{Z}_2)$, $\mathbb{V}(\mathrm{Z}_3)$, $\mathbb{V}(\mathrm{Z})$, *and* $\mathbb{Q}(\mathrm{E})$ *has the amalgamation property.*

Proof sketch

1. Due to Czelakowski-Dziobiak characterization all of the four (quasi)varieties have RCEP.
2. In each of the four cases, finitely generated RFSIs are just FSIs, i.e. Sugihara chains. Chains are closed under subalgebras.
3. [Gil-Férez et al., 2020] for any Sugihara chains, there is an amalgam (linear odd Sugihara monoid).
4. Apply [Fussner and Metcalfe, 2024] to conclude that all of the four classes have AP.

The challenging part– negative part

"Any other quasivariety of Sugihara algebras does not have AP"

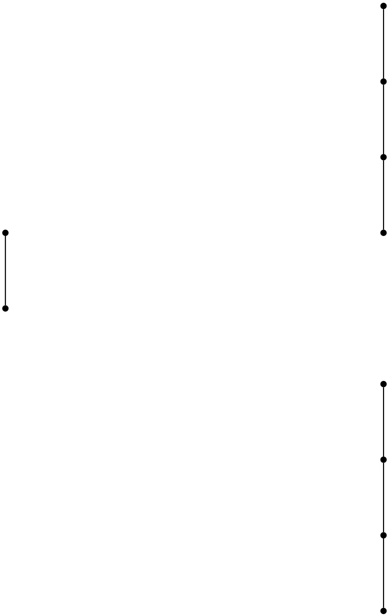
or rather

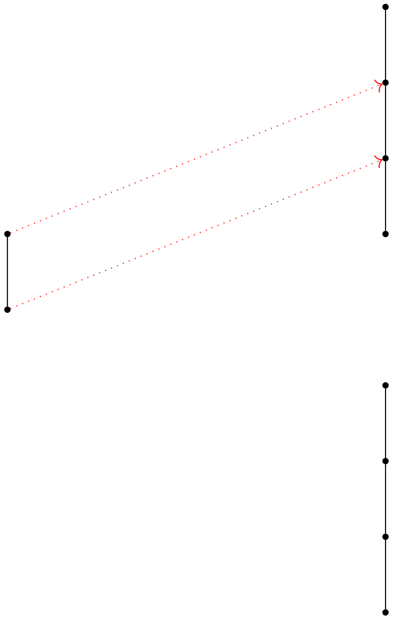
"If a quasivariety of Sugiharas has AP, then it is one of the five from the list"

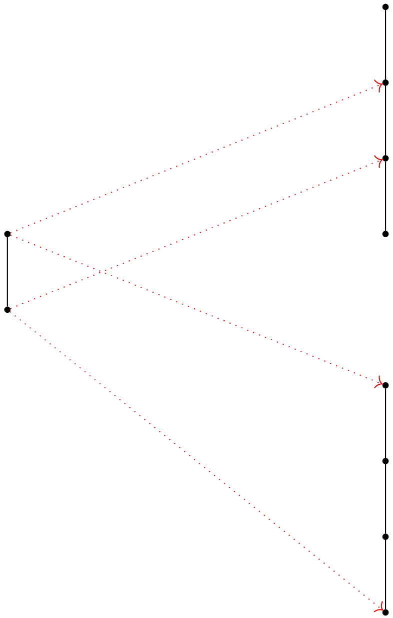
The two extending lemmas (even case)

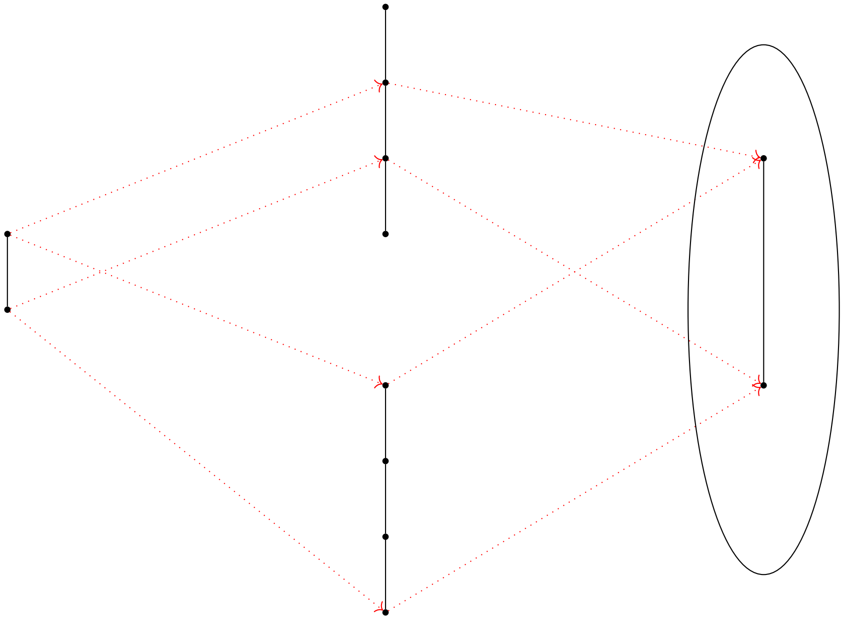
Lemma

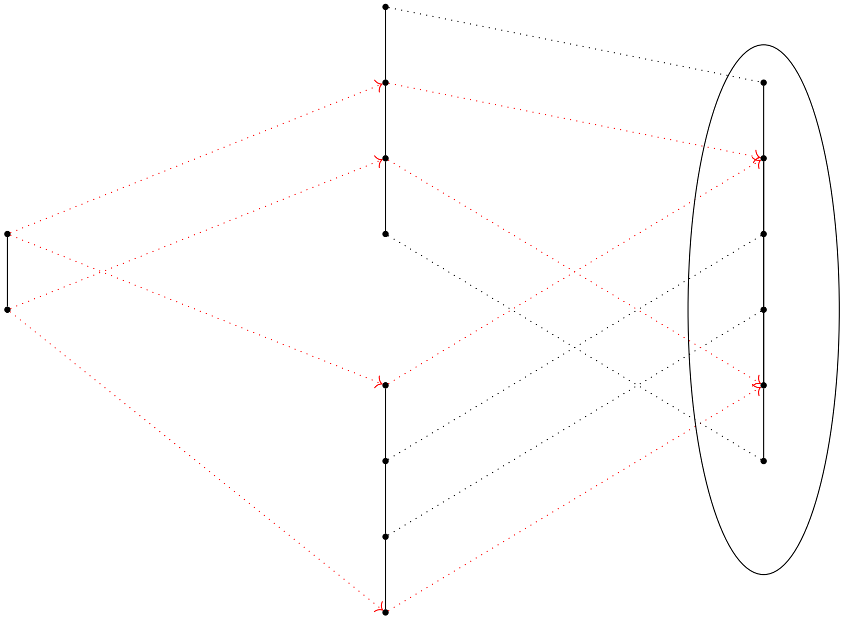
Assume Q *has AP. If* $\mathsf{Z}_4 \in \mathsf{Q}$*, then* $\mathsf{Z}_{2n} \in \mathsf{Q}$ *for every positive integer n. Consequently, if* $\mathsf{Z}_4 \in \mathsf{Q}$*, then* $\mathsf{E} \in \mathsf{Q}$*.*











The two extending lemmas (odd case)

Lemma

Assume Q *has AP. If* $\mathsf{Z}_3, \mathsf{Z}_4 \in \mathsf{Q}$, *then* $\mathsf{Q} = \mathsf{SA}$.

"The coordinate switch" embedding – odd case

Lemma

Let Q *has AP. If* $\mathsf{Z}_2 \times \mathsf{Z}_3 \in \mathsf{Q}$*, then* $\mathsf{Z}_3 \in \mathsf{Q}$*.*

Sketch of an argument.

Take a span $\langle i \colon \mathsf{Z}_2 \times \mathsf{Z}_2 \hookrightarrow \mathsf{Z}_2 \times \mathsf{Z}_3, e \colon \mathsf{Z}_2 \times \mathsf{Z}_2 \hookrightarrow \mathsf{Z}_2 \times \mathsf{Z}_3 \rangle$ where i is the identity embedding and e is the 'coordinate-switch' embedding $(n, m) \mapsto (m, n)$.

$$
\begin{tikzcd}
& \mathsf{Z}_2 \times \mathsf{Z}_3 \arrow[dr, hook, "i'"] & & \\
\mathsf{Z}_2 \times \mathsf{Z}_2 \arrow[ur, hook, "i"] \arrow[dr, hook, "e"'] & & A & \mathsf{Z}_3 \in Q \arrow[l, hook'] \\
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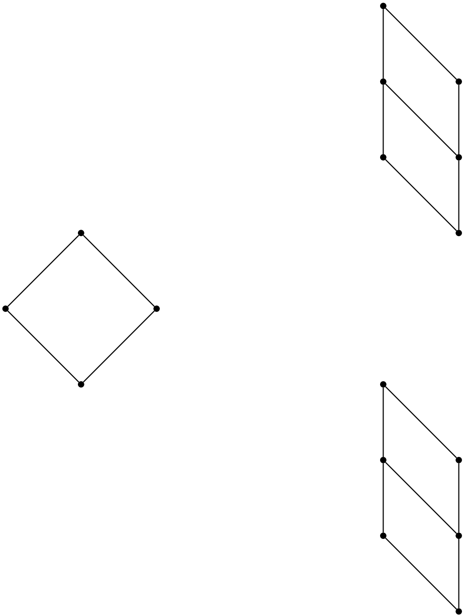

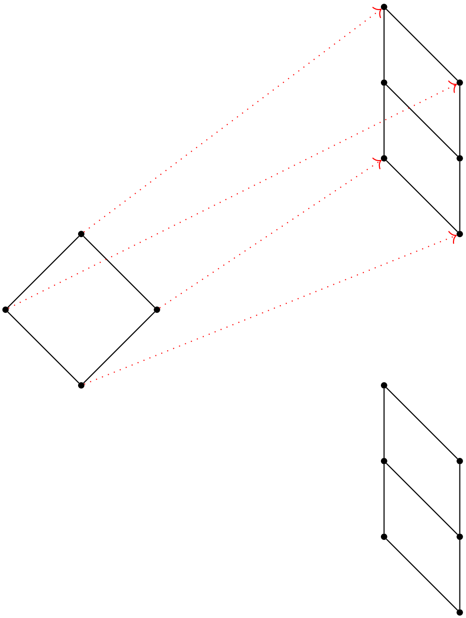

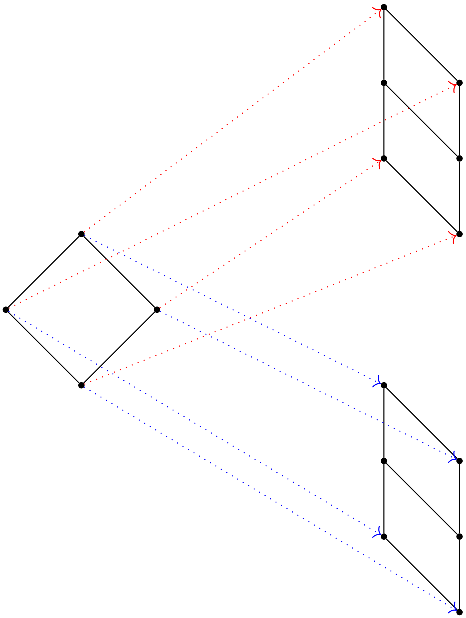

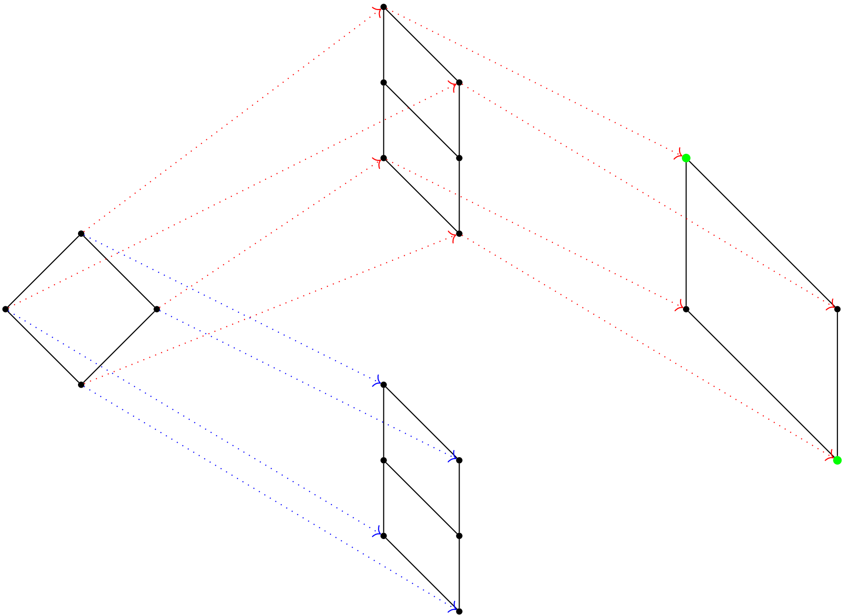

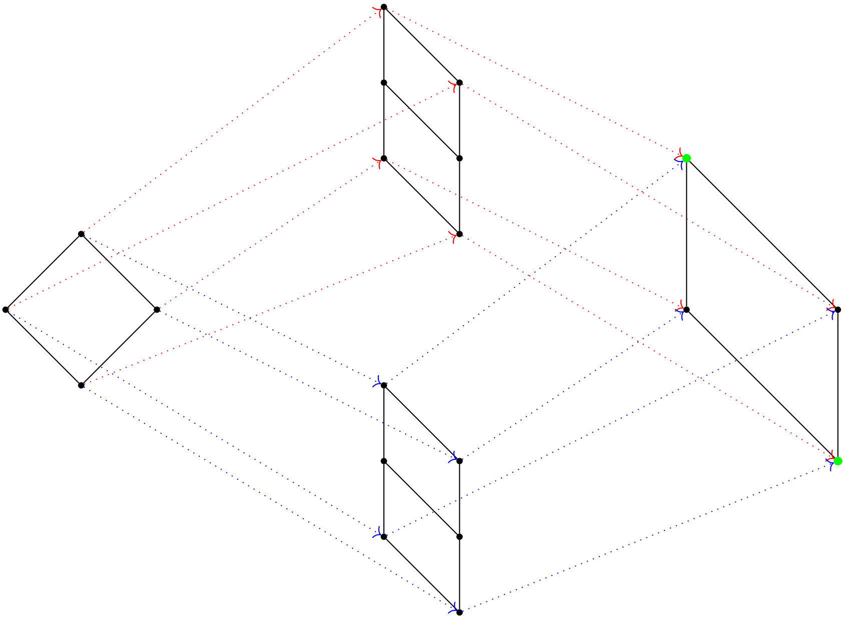

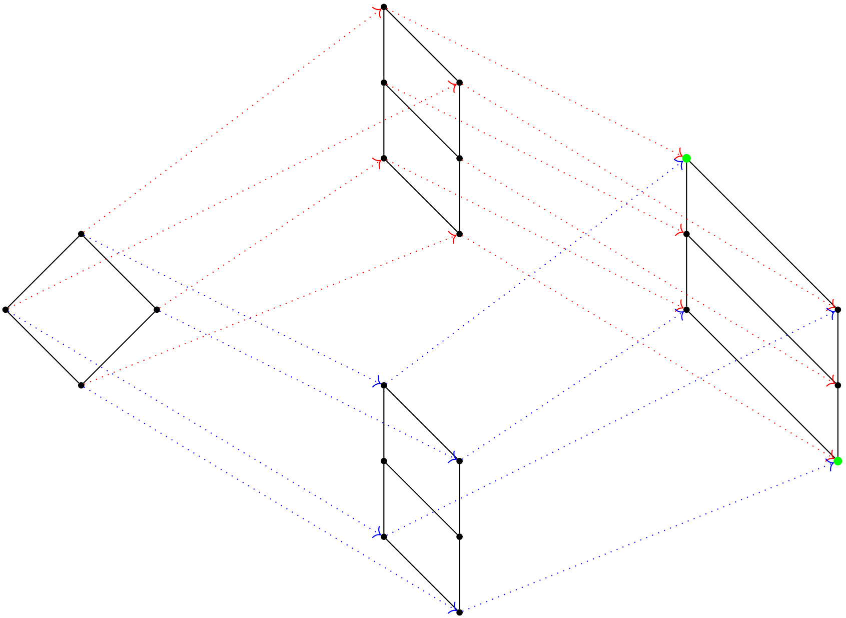

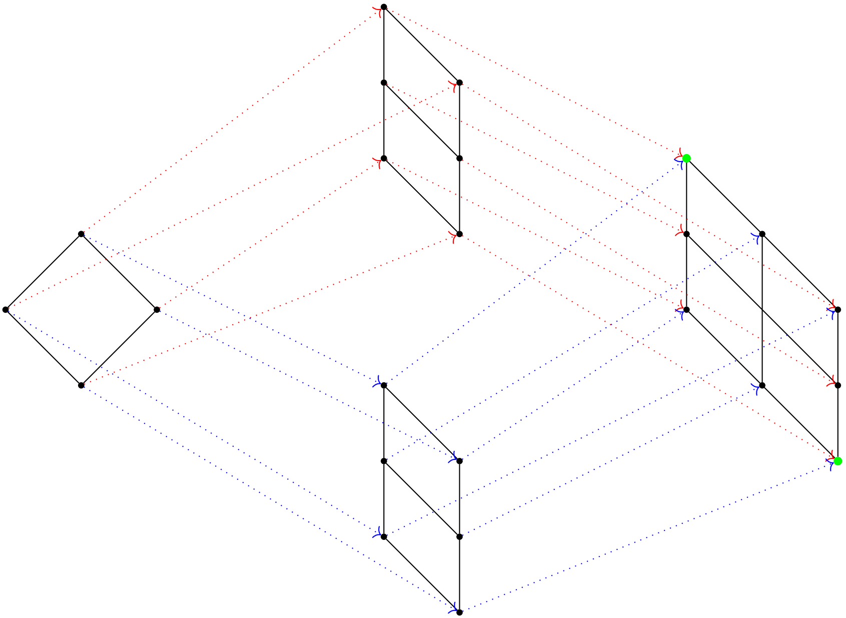

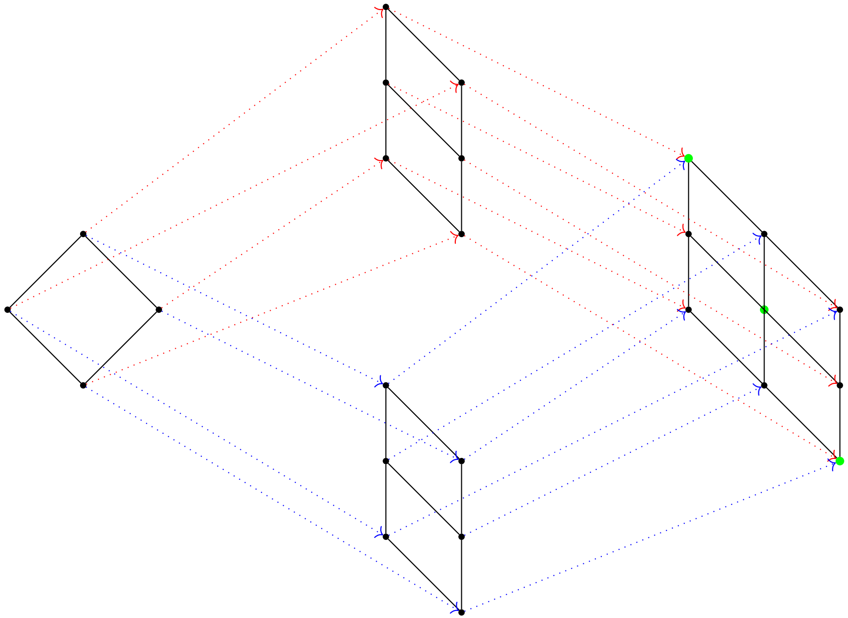

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\end{tikzcd}$$

Theorem

Assume a nontrivial subquasivariety of SA, Q has AP. Then it is one of the four: $\mathbb{V}(Z_2)$, $\mathbb{V}(Z_3)$, $\mathbb{V}(Z)$, and $\mathbb{Q}(E)$.

proof

Since Q is nontrivial, $\mathbb{V}(Z_2) = \mathbb{Q}(Z_2) \subseteq Q$. Assume that this containment is proper. Then, by Lemma (KK2022), either $Z_2 \times Z_3 \in Q$ or $Z_2 \times Z_4 \in Q$. We consider three mutually exclusive cases.

proof

First, suppose that both $Z_2 \times Z_3 \in Q$ and $Z_2 \times Z_4 \in Q$. Then $Z_3, Z_4 \in Q$ by the two "coordinate-switch" lemmas. It then follows by the "chain-extending" Lemma that $Q = SA = \mathbb{V}(Z)$.

proof

Second, suppose that $Z_2 \times Z_3 \in Q$ and $Z_2 \times Z_4 \notin Q$. Then, since $Z_2 \times Z_3 \in Q$, applying the first "coordinate-switch" Lemma gives that $Z_3 \in Q$. Therefore, $\mathbb{V}(Z_3) = \mathbb{Q}(Z_3) \subseteq Q$.

Lemma

Let Q be a quasivariety that properly contains $\mathbb{V}(Z_3)$. Then $Z_2 \times Z_4 \in Q$.

On the other hand, since $Z_2 \times Z_4 \notin Q$, the above Lemma gives us that $\mathbb{V}(Z_3)$ is not properly contained in Q, i.e., $Q = \mathbb{V}(Z_3)$.

proof

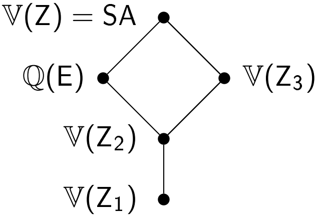
Third, suppose that $Z_2 \times Z_3 \notin Q$ and $Z_2 \times Z_4 \in Q$. Since $Z_2 \times Z_4 \in Q$, applying "coordinate-switch" lemma and "extending" lemma gives that $E \in Q$ and, hence, $\mathbb{Q}(E) \subseteq Q$.

Lemma

Let $\mathbb{Q}(E) \subsetneq Q$. Then $Z_2 \times Z_3 \in Q$.

By the above Lemma $Q = \mathbb{Q}(E)$, and the result follows.

Poset of Qs with AP



Immediate Corollarry

In quasivarieties of Sugihara algebras $AP \implies RCEP$. (Opposite direction is false) and additionally $AP \Leftrightarrow TIP$ (TIP always implies AP, but not the other way around)

R-mingle and Sugihara algebras

The logic R-mingle results from adding the 'mingle axiom' to the basic system of relevance logic R:

$$\alpha \to (\alpha \to \alpha)$$

RM is algebraizable w.r.t. the variety of Sugihara algebras.

subvarieties correspond to axiomatic extensions and subquasivarieties to arbitrary consequence relations extending RM.

If $\mathrm{var}(\alpha) \cap \mathrm{var}(\beta) \neq \emptyset$, and $\vdash \alpha \to \beta$, then there is a formula δ such that $\mathrm{var}(\delta) \subseteq \mathrm{var}(\alpha) \cap \mathrm{var}(\beta)$ and both $\vdash \alpha \to \delta$ and $\vdash \delta \to \beta$. (CIP)

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If $\Gamma \vdash \beta$ and $\mathrm{var}(\Gamma) \cap \mathrm{var}(\beta) \neq \emptyset$, then there is a set of formulas Δ such that $\mathrm{var}(\Delta) \subseteq \mathrm{var}(\Gamma) \cap \mathrm{var}(\beta)$ and both $\Gamma \vdash \Delta$ and $\Delta \vdash \beta$. (DIP)

AAL – false bridge theorem

CIP $\not\Leftrightarrow$ AP $\not\Leftrightarrow$ DIP

AP $\Leftrightarrow$ RP [Metcalfe et al., 2014]

$$\text{Whenever } X, Y \subseteq \text{var such that } X \cap Y \neq \emptyset,\ T \text{ is a theory of } \mathrm{L} \text{ over } X, \text{ and } S \text{ is a theory of } \mathrm{L} \text{ over } Y \text{ such that } T \cap \mathrm{Fm}_{\mathcal{L}}(X \cap Y) = S \cap \mathrm{Fm}_{\mathcal{L}}(X \cap Y), \text{ there exists a theory } R \text{ of } \mathrm{L} \text{ over } X \cup Y \text{ such that } T = R \cap \mathrm{Fm}_{\mathcal{L}}(X) \text{ and } S = R \cap \mathrm{Fm}_{\mathcal{L}}(Y). \tag{RP}$$

AAL bridge theorem: Robinson property

If $\mathsf{var}(\Sigma \cup \{\alpha\}) \cap \mathsf{var}(\Gamma) \neq \emptyset$ and $\Sigma, \Gamma \vdash \alpha$, there exists a set of formulas Δ such that $\mathsf{var}(\Delta) \subseteq \mathsf{var}(\Sigma \cup \{\alpha\}) \cap \mathsf{var}(\Gamma)$ and both $\Gamma \vdash \Delta$ and $\Sigma, \Delta \vdash \alpha$. (MIP)

MIP $\Leftrightarrow$ TIP $\Leftrightarrow$ AP + RCEP [Czelakowski and Pigozzi, 1999].

Conclusion

Exactly five consequence extensions of RM (out of how many??) have the MIP and only one of them is non-axiomatic.

MIP and RP coincide for extensions of RM.

Thank you!

W. Fussner, K. Krawczyk, *Maehara Interpolation in Extensions of R-mingle*, https://doi.org/10.48550/arXiv.2512.04762

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