

- **Russel's paradox:**  $A = \{x : x \notin x\}$ . Is  $A \in A$ ?
- **Hypergame paradox:** A two person game is called well-founded if it always ends in a finite number of steps. The Hypergame is the two player game with these rules: player  $A$  starts the game by selecting a well-founded game; player  $B$  starts playing the well-founded game chosen by  $A$ ; and then the players play the selected well-founded game until its end. Is the Hypergame well-founded?
- **Grelling's paradox:** Some adjectives apply to themselves: "short" is short, "polysyllabic" is polysyllabic, "English" is English. These adjectives are called autological. Some adjectives do not apply to themselves: "green" is not green "long" is not long. These adjectives are called heterological. Every adjective is either heterological or it is autological. Is "heterological" heterological or autological?
- **Berry paradox:** The number of integers that can be named in English in less than a fixed, finite (70, say) number of letters is finite. So the set  $X$  of integers that cannot be named by such sentences is not empty and so  $X$  contains a least element. So there is a least number  $n$  that cannot be named by a sentence containing 70 letters. But the sentence:

"The least integer that cannot be named in English in less than 70 letters"

is a sentence that contains less than 70 (in fact: 60) letters and names the number  $n$ . Contradiction.

- **Paradox of the question:** During a large and international conference of the world's leading philosophers, an angel miraculously appeared and said: I come to you as a messenger from God. You will be permitted to ask any one question you want - but only one! - and I will answer that question truthfully. What would you like to ask?

Suggestion no. 1.: If I eat myself, would I disappear or would I become twice as big?

Suggestion no. 2: What is the answer to the question that would be the best question for us to ask? (Suggestion 2 rejected on the grounds that an answer such as "Yes" or "Seven" is uninformative unless one knows the question).

Suggestion no.3: What is the ordered pair whose first member is the question that would be the best one for us to ask you, and whose second member is the answer to that question?

Answer: It is the ordered pair whose first member is the question you just asked me, and whose second member is this answer I am giving you.

- **Law of excluded middle.** The story goes:
  - Amy said you didn't go to school yesterday.
  - She was wrong about it though!
  - So you did go to school?
  - What makes you say that?

On the other hand:

**Proposition 0.1.** There exist two (not necessarily different) irrational real numbers  $a, b$  such that  $a^b$  is a rational number.

**Proof.** We know that  $\sqrt{2}$  is irrational. Consider  $\sqrt{2}^{\sqrt{2}}$ . If it is rational, then end of proof. If it is irrational, then  $(\sqrt{2}^{\sqrt{2}})^{\sqrt{2}} = (\sqrt{2})^{\sqrt{2}\sqrt{2}} = (\sqrt{2})^2 = 2$  is rational, end of proof. ■