Axioms:

Ax.1.	$A \to (B \to B)$	Ax.7.	$(A \to B) \to ((A \to C) \to (A \to (B \wedge C)))$
Ax.2.	$(A \to B) \to ((B \to C) \to (A \to C))$	Ax.8.	$A \to (A \vee B)$
Ax.3.	$(A \to (B \to C)) \to (B \to (A \to C))$	Ax.9.	$B \to (A \vee B)$
Ax.4.	$(A \to (A \to B)) \to (A \to B)$	Ax.10.	$(A \to C) \to ((B \to C) \to ((A \vee B) \to C))$
Ax.5.	$(A \wedge B) \to A$	Ax.11.'	$(A \to B) \to ((A \to \sim B) \to \sim A)$
Ax.6.	$(A \wedge B) \to B$	Ax.12.	$A \to (\sim A \to B)$

Inference rule(s):

RO=Modus ponens	**RKom**	**RP**	**RPI**	**RPO**
$\dfrac{\alpha \qquad \alpha \to \beta}{\beta}$	$\dfrac{\alpha \to (\beta \to \gamma)}{\beta \to (\alpha \to \gamma)}$	$\dfrac{\alpha}{\beta \to \alpha}$	$\dfrac{\alpha \to \beta \qquad \beta \to \gamma}{\alpha \to \gamma}$	$\dfrac{\alpha \to (\beta \to \gamma) \qquad \alpha \to \beta}{\alpha \to \gamma}$

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1. Check that the KRZ derivations of the following formulas are intuitionistic derivations as well (handout #6).

$$\vdash_{int} \alpha \to \alpha \qquad \vdash_{int} \alpha \to (\beta \to \alpha) \qquad \alpha \to \beta,\ \beta \to \gamma \vdash_{int} \alpha \to \gamma$$

2. Construct derivations in INT for the following formulas.
   (a) $\alpha \to \sim\sim\alpha$ however: $\nvdash_{int} \sim\sim\alpha \to \alpha$
   (b) $(\alpha \to \beta) \to (\sim\beta \to \sim\alpha)$ however: $\nvdash_{int} (\sim\alpha \to \sim\beta) \to (\beta \to \alpha)$
   (c) $(\alpha \to \sim\alpha) \to \sim\alpha$ however: $\nvdash_{int} (\sim\alpha \to \alpha) \to \alpha$
   (d) $\sim(\alpha \wedge \sim\alpha)$ however: $\nvdash_{int} \alpha \vee \sim\alpha$

Hints:

(1/a) Ax.1 with $A = \alpha \to (\beta \to \beta)$, $B = \alpha$, then Ax.1 with $A = \alpha, B = \beta$.

(1/b) Ax.1 with $A = \beta, B = \alpha$, then Ax.3 with $A = \beta$, $B = \alpha$, $C = \alpha$.

(1/c) Ax.2

(2/a) Ax.11' ($A = \sim\alpha$, $B = \alpha$); (1/b): $\beta / \sim\alpha$; RPI: 2,1; RKom: 3; (1/a): $\alpha / \sim\alpha$; RO: 4,5.

(2/b) Ax.11'; RKom: 1; (1/b): $\alpha / \sim\beta, \beta/\alpha$; RPI: 3,2; RKom: 4.

(2/c) Ax.11' ($A = \alpha, B = \alpha$); (1/a); RO: 1,2.

(2/d) Ax.11' ($A = \alpha \wedge \sim\alpha$, $B = \alpha$); Ax.5 ($A = \alpha$, $B = \sim\alpha$); RO: 1,2; Ax.6 ($A = \alpha$, $B = \sim\alpha$); RO: 3,4.