Definition: $\Gamma \models \varphi$ (φ is a semantic consequence of Γ) if and only if whenever an evaluation makes all formulas of Γ true, then it makes φ true as well.

$$\text{FOR ALL EVALUATIONS } e, \text{ IF } e(\Gamma) = 1, \text{ THEN } e(\varphi) = 1$$

Deduction theorem of KRZ: $\psi \models \phi$ if and only if $\models \psi \to \phi$. We reduced the notion of consequence to that of tautology.

1. Check if the following formulas are tautologies: $(p \wedge (p \to q)) \to p$, $(p \wedge (p \to q)) \to q$.
2. Verify that $\{A \to B, B \to C\} \models A \to C$.
3. Formalize the sentences below and check whether any of the consequences hold.
   - If John solves less than half of the exercises, then he fails the exam.
   - He is either cheating or cannot solve at least half of the exercises.
   - (Consequence#1?) If he is cheating, then he passes the exam.
   - (Consequence#2?) If he passes the exam, then he is cheating.
4. Formalize the following arguments and verify whether they are correct.

   If Charles won the competition, then either Mark came second or Sean came third. Sean didn't come third. Thus, if Mark didn't come second, then Charles didn't win the competition.

   If you play and you study you'll pass the exams, while if you play and don't study you won't pass. Thus, if you play, either you study and you'll pass the exams, or you don't study and you won't pass.
5. Aladdin finds two trunks A and B in a cave. He knows that each of them either contains a treasure or a fatal trap.

   On trunk A is written: "At least one of these two trunks contains a treasure."
   On trunk B is written: "In A there's a fatal trap."

   Aladdin knows that either both the inscriptions are true, or they are both false. Can Aladdin choose a trunk being sure that he will find a treasure? If this is the case, which trunk should he open?

HW #1 Suppose we know that:

- if Paul is thin, then Charles is not blonde or Jane is not tall
- if Jane is tall then Sandra is lovely
- if Sandra is lovely and Charles is blonde then Paul is thin
- Charles is blonde

Can we deduce that Jane is not tall?

HW #2 Three boxes are presented to you. One contains gold, the other two are empty. Each box has imprinted on it a clue as to its contents; the clues are:

- Box 1: "The gold is not here"
- Box 2: "The gold is not here"
- Box 3: "The gold is in Box 2"

Only one message is true, the other two are false. Which box has the gold?

Formalize the puzzle in propositional logic and find the solution using a truth table.

SOLUTIONS

4. Let C="Charles won", M="Mark came second", S="Sean came third". Then

$$C \to (M \vee S),\ \sim S \quad \overset{?}{\models} \quad (\sim M) \to (\sim C)$$

Let P="play", S="study" and E="pass the exam". Then

$$(P \wedge S) \to E,\ (P \wedge \sim S) \to \sim E \quad \overset{?}{\models} \quad P \to ((S \to E) \vee (\sim S \to \sim E))$$

5. Consider a propositional language where $a =$"A contains the treasure", $b =$"B contains the treasure". Then $\sim a$="A contains a trap", $\sim b$="B contains a trap".

Aladdin knows:

- $a \vee b$ "At least one of these two trunks contains a treasure"
- $\sim a$ "A contains a trap"

Formalization of the problem:

- $(a \vee b) \leftrightarrow \sim a$ "either both the inscriptions are true, or they are both false"

Is there an evaluation that satisfies the formula $(a \vee b) \leftrightarrow \sim a$? The only such evaluation is $v(a) = 0$, $v(b) = 1$. Thus Aladdin can open trunk B, being sure that it contains a treasure.

HW 1. With $P =$"Paul is thin", $C =$"Charles is blonde", $J =$"Jane is tall", $S =$"Sandra is lovely", the question is

$$P \to (\sim C \vee \sim J),\ J \to S,\ (S \wedge C) \to P,\ C \quad \overset{?}{\models} \quad \sim J$$

HW 2. Let $B_i =$"the gold is in the i^{th} box". We can formalize the statements as

- One box contains gold, the other two are empty:

$$(B_1 \wedge \sim B_2 \wedge \sim B_3) \vee (\sim B_1 \wedge B_2 \wedge \sim B_3) \vee (\sim B_1 \wedge \sim B_2 \wedge B_3)$$

- Only one message is true; the other two are false.

$$(\sim B_1 \wedge \sim\sim B_2 \wedge \sim B_2) \vee (\sim\sim B_1 \wedge \sim B_2 \wedge \sim B_2) \vee (\sim\sim B_1 \wedge \sim\sim B_2 \wedge B_2)$$

This latter is equivalent to $(B_1 \wedge \sim B_2) \vee (B_1 \wedge B_2)$.

Computing the truth tables of the formulas above, the only evaluation that verifies both statements is $v(B_1) = 1$, $v(B_2) = v(B_3) = 0$. Thus, the gold is in the first box.