Axioms:

Ax.1. $A \to (B \to B)$

Ax.2. $(A \to B) \to ((B \to C) \to (A \to C))$

Ax.3. $(A \to (B \to C)) \to (B \to (A \to C))$

Ax.4. $(A \to (A \to B)) \to (A \to B)$

Ax.5. $(A \wedge B) \to A$

Ax.6. $(A \wedge B) \to B$

Ax.7. $(A \to B) \to ((A \to C) \to (A \to (B \wedge C)))$

Ax.8. $A \to (A \vee B)$

Ax.9. $B \to (A \vee B)$

Ax.10. $(A \to C) \to ((B \to C) \to ((A \vee B) \to C))$

Ax.11. $(\sim A \to \sim B) \to (B \to A)$

Inference rule: $A,\ A \to B\ \vdash\ B$ (RO) (Modus Ponens)

Deduction: $\Phi, \alpha\ \vdash\ \beta$ if and only if $\Phi\ \vdash\ \alpha \to \beta$

Theorem: $\Phi\ \vdash\ \alpha$ if and only if $\Phi \cup \{\sim\alpha\}$ is contradictory.

Similarly: $\Phi\ \vdash \sim\alpha$ if and only if $\Phi \cup \{\alpha\}$ is contradictory.

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1. Show that $\alpha \to (\beta \to \gamma) \vdash \beta \to (\alpha \to \gamma)$. (RKom)
2. Show that $\alpha \vdash \beta \to \alpha$ (Hint: use Ax.1 and Ax.2)
3. Derive
   - $\alpha, \alpha \to \beta \vdash \beta$
   - $(\alpha \to \beta) \to (\alpha \to \gamma), \alpha, \beta \vdash \gamma$
   - $\alpha \to \beta, \sim\beta, \alpha \vdash$ contradiction
4. Assume $\vdash \varphi \to \psi$ and $\vdash \psi \to \varphi$. Is it true in this case that $\vdash \varphi$ or $\vdash \psi$?
5. Show $\alpha \to \beta,\ \beta \to \gamma\ \vdash\ \alpha \to \gamma$ using the deduction theorem.
6. Show that $\alpha \to \beta, \gamma\ \nvdash\ \beta$.
7. Prove that if $\Sigma \vdash \varphi$ then there is a finite subset $\Gamma \subseteq \Sigma$ such that $\Gamma \vdash \varphi$.
8. Show $\alpha \to \beta, \sim\beta\ \vdash \sim\alpha$
9. Show that if $\alpha \vdash \varphi$ and $\sim\alpha \vdash \varphi$, then φ is a theorem (i.e. $\vdash \varphi$).
10. Is it true that if $\neg\alpha \vdash \beta$ then $\neg\beta \vdash \alpha$?
11. Assume $\alpha \vdash \beta$ and $\vdash \alpha$. Show that $\vdash \beta$.