To say this is not to say that the axiom of choice is not both obvious and indispensable. It is only to say that the justification for its acceptance is not to be found in the iterative conception of set.

GEORGE BOOLOS

Massachusetts Institute of Technology

A PLEA FOR SUBSTITUTIONAL QUANTIFICATION *

IN this note I shall discuss the relevance to ontology of what is called the *substitutional* interpretation of quantifiers. According to this interpretation, a sentence of the form '$(\exists x)Fx$' is true if and only if there is some closed term 't' of the language such that 'Ft' is true. This is opposed to the *objectual* interpretation according to which '$(\exists x)Fx$' is true if and only if there is some object x in the universe of discourse such that 'F' is *true of* that object.[1]

Ontology is not the only connection in which substitutional quantification has been discussed in recent years. It has been advocated as a justification for restrictions on the substitutivity of identity in intensional languages[2] or has been found necessary to

* I owe to Sidney Morgenbesser and W. V. Quine the stimulus to write this paper. I am also indebted to Hao Wang for a valuable discussion, and to Quine for illuminating comments on an earlier version.

[1] It is noteworthy that the substitutional interpretation allows truth of, say, first-order quantified sentences to be given a direct inductive definition, while in the objectual interpretation the fundamental notion is truth *of* (satisfaction), and truth is defined in terms of it. Davidson's version of the correspondence theory of truth would not be applicable to a substitutional language. See "True to the Facts," this JOURNAL, LXVI, 21 (Nov. 6, 1969): 748–764.

Quine has pointed out to me that in a language with infinitely many singular terms, a problem arises about defining truth for *atomic* formulas. Objectual quantification can be nontrivial for a language with no singular terms at all, but, if there are such, the problem can be resolved by an auxiliary definition which assigns them denotations, relative to a sequence that codes an assignment to the free variables. In the substitutional case, we need to define the truth of an atomic formula $Pt_1 \ldots t_m$ directly for closed terms $t_1 \ldots t_m$, for example, inductively by reducing the case for more complex terms to that for less complex terms. In the usual language of elementary number theory, closed terms are constructed from '0' by applications of various function symbols which (except for the successor symbol, say 'S') have associated with them defining equations that can be regarded as rules for reducing closed terms to canonical form, namely, as numerals (a numeral is 'S' applied finitely many times to '0'), so that an atomic formula $Pt_1 \ldots t_m$ is true if and only if $Pn_1 \ldots n_m$ is true, where n_i is the numeral corresponding to t_i. The truth of predicates applied to numerals is defined either trivially (as in the case of '$=$') or in a similar recursive manner.

Quine discusses this issue in §6 of an unpublished paper, "Truth and Disquotation."

[2] Ruth Barcan Marcus, "Modalities and Intensional Languages," *Synthese*, XIII,

make sense of restrictions adopted in certain systems.[3] With this matter I shall not be concerned.

It might seem that in discussions of ontology the substitutional interpretation of quantifiers would be advocated in order to make acceptable otherwise questionable ontological commitments. In fact this has not been done widely, although it does seem to be an important part of Wilfrid Sellars' account of abstract entities.[4] The issue of the ontological relevance of substitutional quantification has been raised most explicitly by Quine, essentially to debunk it. He argues that only an interpretation of a theory in terms of *objectual* quantification attributes an ontology to it.

Classical first-order quantification theory on the objectual interpretation, according to Quine, embodies the fundamental concept of existence. He acknowledges the existence of other possible concepts of existence. But he holds that substitutional quantification does not embody a genuine concept of existence at all.

I should like to argue that the existential quantifier substitutionally interpreted has a genuine claim to express a concept of existence which has its own interest and which may offer the best explication of the sense in which "linguistic" abstract entities—propositions, attributes, classes in the sense of extensions of predicates[5]—may be said to exist. I shall then raise a difficulty for the view (which

4 (December 1961): 303–322. Reprinted in I. M. Copi and J. A. Gould, eds., *Contemporary Readings in Logical Theory* (New York: Macmillan, 1967), pp. 278–293. Cf. W. V. Quine, "Reply to Professor Marcus," *Synthese, loc. cit.*: 323–330; reprinted in Copi and Gould, pp. 293–299; also in *The Ways of Paradox* (New York: Random House, 1966), pp. 175–182.

[3] Particularly Hintikka's. See Dagfinn Føllesdal, "Interpretation of Quantifiers," in B. van Rootselaar and J. F. Staal, eds., *Logic, Methodology, and Philosophy of Science* III (Amsterdam: North Holland, 1968), pp. 271–281, and papers by Hintikka and Føllesdal referred to there.

[4] "Abstract Entities," *Review of Metaphysics*, XVI, 4 (June 1963): 627–671, reprinted in *Philosophical Perspectives* (Springfield, Ill.: Charles C. Thomas, 1967), pp. 229–69. Sellars' general strategy is to treat attributes and classes as analogous to linguistic types and then to quantify substitutionally over them. This seems to be open to the objection presented below. Sellars' account of classes seems to yield a predicative theory of classes and thus would not justify set theory. See "Classes as Abstract Entities and the Russell Paradox," *Review of Metaphysics*, XVII, 1 (September 1963): 67–90; also *Philosophical Perspectives*, pp. 270–290. In studying these two papers I have relied heavily on Gilbert Harman's lucid review of *Philosophical Perspectives*, this JOURNAL, LXVI, 5 (March 13, 1969): 133–144.

[5] This account of classes would suggest distinguishing *classes* in this sense from *sets* as the objects of set theory. One can, I believe, motivate the requirement of predicativity for the former. It is they which are paralleled to attributes. It is noteworthy that there is no reason to make impredicative assumptions about the existence of attributes, unless (as in Quine's interpretation of Russell's no-class theory) one seeks to reduce sets to attributes.

Sellars may hold) that all quantification over abstract entities can be taken to be substitutional.

Quine argues as follows for the view that substitutional quantification does not correspond to a genuine concept of existence:

> Substitutional quantification makes good sense, explicable in terms of truth and substitution, no matter what substitution class we take —even that whose sole member is the left-hand parenthesis. To conclude that entities are being assumed that trivially, and that far out, is simply to drop ontological questions. Nor can we introduce any control by saying that only substitutional quantification in the substitution class of singular terms is to count as a version of existence. We just now saw one reason for this, and there is another: the very notion of singular terms appeals implicitly to classical or objectual quantification.[6]

In answer to the first objection, we should point out two formal features of the category of singular terms that mark substitutional quantification with respect to it as far less trivial than with respect to, say, the left parenthesis. First, it admits identity with the property of substitutivity *salva veritate*. Second, it has infinitely many members that are distinguishable by the identity relation. This has the consequence that '$(\forall x)Fx$' is stronger than any conjunction that can be formed of sentences of the form 'Ft', while '$(\exists x)Fx$' is weaker than any disjunction of such sentences.

With respect to the claim that the very notion of singular terms appeals implicitly to classical or objectual quantification, we might hope for a purely syntactical characterization of singular terms. However, that would not yet yield the distinction between singular terms that genuinely refer and those which do not; in a language in which the latter possibility arises, the substitutional quantifier for singular terms would express not existence but something closer to Meinong's "being an object."

However, we can concede Quine's point here for a certain central core class of singular terms, which we might suppose to denote objects whose existence we do not expect to explicate by substitutional quantification. We might then make certain analogical extensions of the class of singular terms in such a way that they are related to quantifications construed as substitutional. The criterion for "genuine reference" is given in other terms.

For example, the following is a natural way to introduce a predica-

[6] *Ontological Relativity and Other Essays* (New York: Columbia, 1969), p. 106, Cf. pp. 63–64.

tive theory of classes (extensions of predicates). Let 'F' stand for a one-place predicate of some first-order language. We first rewrite 'Ft' as '$t\epsilon\{x:Fx\}$' and (taking 'α', 'β', ... as schematic letters for expressions of the form '$\{x:Fx\}$') define '$\alpha=\beta$' as '$(\forall x)(x\epsilon\alpha \equiv x\epsilon\beta)$'. So far we have just made the contextual definitions involved in the theory of virtual classes.[7] We then allow the abstracts to be replaced by quantifiable variables of a new sort. The substitution interpretation gives truth conditions for formulas in the enlarged notation. The process can be repeated to introduce classes of higher levels.[8]

The advantage of substitutional quantification in this particular case is that it fits the idea that the classes involved are not "real" independently of the expressions for them. More precisely, we know the condition for a predicate to "have an extension" (that it be true or false of each object in the universe) and for two predicates to "have the same extension" without independently identifying the extension. The fact that the substitution interpretation yields truth conditions for quantified sentences means that everything necessary for speaking of these classes as entities is present, and the request for some more absolute verification of their existence seems senseless.

The obstacle to the introduction of attributes in the same way is, of course, the problem of the criterion of identity. But the procedure goes through, given a suitable intensional equivalence relation. For example, we might introduce "virtual attributes" by rewriting 'Ft' as '$t\,\delta\,[x:Fx]$', introducing 'ξ', 'η' ... as schematic letters, defining '$\xi = \eta$' as '$\Box(\forall x)(x\,\delta\,\xi \equiv x\,\delta\,\eta)$', and then introducing attribute variables and substitutional quantification. Then two predicates express the same attribute provided they are necessarily coextensive.

The same procedure could be followed for other intensional equivalences, provided that they can be expressed in the object language.

Consideration of examples such as these leads to the conclusion that in the case where the terms involved have a nontrivial equivalence relation with infinitely many equivalence classes, substitutional quantification gives rise to a genuine "doctrine of being" to be set alongside Quine's and others. It parallels certain idealistic theories

[7] Quine, *Set Theory and Its Logic* (Cambridge, Mass.: Harvard, 2nd ed. 1969), p. 15.

[8] Quine carries out the first stage of this substitutional introduction of classes in *Philosophy of Logic* (Englewood Cliffs, N. J.: Prentice-Hall, 1970), pp. 93–94. If classes are introduced in this way in a language that is not extensional, such as modal logic, then the restrictions on substitutivity of identity associated with a substitutional semantics have point.

of the existence of physical things, such as the account of perception in Husserl's *Ideen.*

It might be thought preferable in our case and perhaps in all cases where the substitution interpretation is workable, to formulate the theory by quantifying over expressions themselves or over Gödel numbers that represent them. If one is talking of expressions or numbers already, this has the advantage of ontological economy, and in some cases, such as when one begins with elementary number theory, it makes more explicit the mathematical strength of the theory.[9] The substitutional approach avoids the artificiality involved in introducing an apparatus (be it Gödel numbering or some other) for talking *about* expressions, and it avoids the unnatural feature that identity of expressions does not correspond to identity of extensions or attributes.

In the case of attributes, however, some proposed criteria of identity, such as synonymy, are metalinguistic. Then the above substitutional introduction of attributes does not apply. Here some form of quantification over linguistic types gives the most natural formulation. An example is Sellars' construal of attributes as synonymity-types of expression-tokens.

The manner in which we have introduced classes suggests a rather arbitrary limitation in the case where the universe for the first-order variables contains unnamed objects. For consider a two-place predicate 'F'. In a theory of virtual classes, we would admit the abstract '$\{x:Fxy\}$' with the free variable 'y', but substitutional quantification as we have explained it encompasses only closed terms. If we wish to say that for every y the class $\{x:Fxy\}$ exists, then the notion of substitutional quantification must be generalized. Suppose a language has variables of two sorts, 'X', 'Y', ... which are substitutionally interpreted, and 'x', 'y' ... which are objectually interpreted over a universe U. Then the fundamental notion (see fn 1) is satisfaction of a formula by a sequence of elements of U. A sequence s satisfies '$(\exists X)FX$' if and only if for some term 'T' of the upper-case sort, with free variables only of the lower-case sort, some extension of s satisfies 'FT'.[10]

In this case we can no longer say that classes are not real independently of expressions for them, but each class is a projection of

[9] See for example the last section of my "Ontology and Mathematics," *The Philosophical Review,* LXXX, 2 (April 1971): 151–176.

[10] The direct construal of classes as expressions fails in this case, and if the universe is of larger cardinality than the set of expressions of the language, classes cannot be so construed even artificially. But of course classes *can* be construed as pairs consisting of an expression and a sequence of substituends for the free variables.

a relation of which this *can* be said. We can say that classes are not real independently of expressions for them and individuals of the universe.

The instances of substitutional quantification we have discussed would suggest that in the process of analogical extension of the category of singular terms, the syntactic characteristics of this category are not all essential. Thus in an extensional language the role of identity can be taken over for one-place predicates by the truth of '$(\forall x)(Fx \equiv Gx)$', and the introduction of 'ramified second-order' substitutional quantifiers seems to differ only notationally from the above introduction of classes.[11] However, such a notational step as the introduction of 'ϵ' and abstracts is necessary if one is to take the further step of reducing the two-sorted theory to a one-sorted one. (The resulting quantifiers would have an interpretation which mixes the substitutional and the objectual.)

In connection with attributes one might be inclined to retreat another step from singular terms and forsake identity. In other words, one might regard quantification over attributes as substitutional quantification of predicates, with no equivalence relation with properties corresponding to those of identity. Here I would agree with Quine that these "attributes" would be at best second-class entities. The ability to get at "the same object" from different points of view—different individual minds, different places and times, different characterizations by language—is one of the essentials of objective knowledge. If this is lacking, then the entities involved should be denied objective existence.

Can all quantification over abstract entities be construed as substitutional? Evidently not if sets as intended by the usual (impredicative) set theories are included. Otherwise we could certainly set up a theory that talked of numbers, of classes in a ramified hierarchy, and even (again in a ramified hierarchy) of propositions

[11] Montgomery Furth in "Two Types of Denotation," *Studies in Logical Theory*, American Philosophical Quarterly Monograph Series no. 2 (Oxford, Blackwell, 1968), pp. 9–45, reconstructs along these lines Frege's idea of predicates as "denoting" concepts rather than objects. He does not remark that on this reconstruction Frege is unjustified in using full (impredicative) second-order logic for quantifying over concepts (more generally, functions). But replacing it by a ramified second-order logic would require him to abandon his Cantorian conception of cardinal number.

Frege's principle that words have meaning (Bedeutung) only in the context of proposition, and his use of it to defend his thesis that numbers are objects, suggests a substitutional account of quantification over "logical objects" such as (on his view) numbers and extensions. Cf. my "Frege's Theory of Number," in Max Black, ed., *Philosophy in America* (London: Allen & Unwin, 1965), pp. 180–203.

and attributes, where the quantifiers over these entities admitted a substitutional interpretation.[12]

Nonetheless there is an obstacle to taking this as implying that nonsubstitutional quantification over abstract entities is avoidable, short of set theory. The difficulty is that the truth conditions for substitutionally quantified sentences themselves involve quantification over expression-types:

'$(\exists x)Fx$' is true if for some *closed term* 't', 'Ft' is true.

Thus the question arises whether the language in which we give this explanation quantifies substitutionally or objectually over expressions. If we give the latter answer, we have of course given up the claim to rely only on substitutional quantification of abstract entities.

In the former case, the same question arises again about the semantics of the metalanguage. We are embarked on a regress that we shall have to end at some point. Then we shall be using a quantifier over expressions that we shall have to either accept as objectual or show in some other way to be substitutional. One might hope to interpret it so that it does not range over abstract entities at all, for example so that there is quantification only over tokens. The ways of doing this that seem to me at all promising involve introducing modality either explicitly or in the interpretation of the quantifier, so that one does not get an ontology of tokens in a Quinean sense.[13]

The only way I can see of showing the quantifier to be substitutional would be to show that the *given* sense of quantification over expression-types in a natural language, say English, is substitutional. This seems to me very implausible.

However, the most this argument would show is that we could not know or prove that the only quantification over abstract entities that we relied on was substitutional. It could still be that an outside observer could interpret our talk in this way. To refute the view that abstract entities (short of set theory) exist only in the substitutional sense, one needs to give a more convincing analysis of such entities as expression-types and numbers, as I have attempted to do elsewhere.[14]

CHARLES PARSONS

Columbia University

[12] Whether in the original sense or in the generalized sense of two paragraphs back would depend on whether the universe of concrete individuals contained unnamed objects. One would suppose the projections referred to above would be needed for some applications.

[13] See section III of "Ontology and Mathematics," cited in fn 9 above.

[14] *Ibid.*