Truth valuational semantics (as in propositional logic) does not work:

1. I am in Kraków.
2. It's possible for me to be in Kraków.
3. 4 is a prime number.
4. It is possible that 4 is a prime number.

Kripke model: (W, R, I) where W = possible worlds, R = accessibility relation, and I = evaluation/interpretation: tells which atomic propositions are true in which worlds. For a world $v \in W$ write $v(\alpha) = 1$ if α is true at v, and $v(\alpha) = 0$ if α is false at v. Rules:

- $v(\neg\alpha) = 1$ iff $v(\alpha) = 0$
- $v(\alpha \wedge \beta) = 1$ iff $v(\alpha) = 1$ and $v(\beta) = 1$
- $v(\alpha \vee \beta) = 1$ iff $v(\alpha) = 1$ or $v(\beta) = 1$
- $v(\alpha \to \beta) = 1$ iff $v(\alpha) = 0$ or $v(\beta) = 1$
- $v(\Diamond\alpha) = 1$ iff there is w such that vRw and $w(\alpha) = 1$
- $v(\Diamond\alpha) = 1$ iff for all w such that vRw, $w(\alpha) = 1$

The formula is **true** (satisfied) **in a model** if it is true at every world.
Modal logic **K**: no restriction on R
Modal logic **S4**: R is transitive and reflexive

1. Check the following formulas on **K** and **S4** models.

1. $\Box p \to p, \qquad p \to \Diamond p, \qquad \Box\Box p \to \Box p, \qquad \Diamond p \to \Diamond\Diamond p.$
2. $\Box p \vee \neg\Box p, \qquad \Box p \vee \Box\neg p$
3. $\Box p \to \Box\neg\neg p$

2. Show by finding countermodels (models where the formulas are refuted).

1. $\not\models_K \neg(\Box\neg p \to \Box(p \to \neg p))$
2. $\not\models_{S4} \neg((\Diamond p \vee \Box q) \vee \neg\Diamond\Diamond p)$
3. $\not\models_{S4} \neg(\neg\Box(\Box p \to \Box q) \to \neg\Box(p \to q))$

3. Translate and find countermodels, if possible.

1. It is possible that it might rain.
2. If Sue runs for office, Louise might run too.
3. We must block door 1 or door 2.
4. To start the engine, the key must be turned.
5. The garbage truck can only lift the bins if they are closed.
6. Sue must not be happy.
7. If parents routinely question their doctor, they might not do what is right for their child.
8. Fred or Mary might have stolen the diamonds, but not both.