

1. Show, by finding countermodels (models where the formulas are refuted).

1. $\not\models_K \neg(\Box\neg p \rightarrow \Box(p \rightarrow \neg p))$
2. $\not\models_{S4} \neg((\Diamond p \vee \Box q) \vee \neg\Diamond\Diamond p)$
3. $\not\models_{S4} \neg(\neg\Box(\Box p \rightarrow \Box q) \rightarrow \neg\Box(p \rightarrow q))$

2. Translate and find countermodels, if possible.

1. It is possible that it might rain.
2. If Sue runs for office, Louise might run too.
3. We must block door 1 or door 2.
4. To start the engine, the key must be turned.
5. The garbage truck can only lift the bins if they are closed.
6. Sue must not be happy.
7. If parents routinely question their doctor, they might not do what is right for their child.
8. Fred or Mary might have stolen the diamonds, but not both.

3. Check the following.

1. $\models_K \Box(p \rightarrow q) \rightarrow (\Box p \rightarrow \Box q)$
2. $\not\models_{S4} (\Box p \rightarrow \Box q) \rightarrow \Box(p \rightarrow q)$
3. $\models_{S4} \Box p \rightarrow \Diamond p$
4. $\not\models_{S4} \Diamond p \rightarrow p$
5. $\not\models_{S4} \Diamond\Box p \rightarrow \Box\Diamond p$
6. $\models_{S4} \Box\Diamond\Box p \rightarrow \Diamond p$