

1. Find models and counter-models for the following sets of formulas.

(a) $\forall x \exists y \exists z (E(x, y) \wedge \neg E(x, z))$;

(b) $\forall x \forall y (x \neq y \rightarrow \exists z (E(x, z) \wedge \neg E(y, z)))$;

(c) $\forall x (\exists y E(x, y) \rightarrow P(x))$;

(d) $\exists x \forall y (f(x, y) = g(y))$.

2. Show that φ is not a consequence of Γ , where

(a) $\Sigma = \{ \forall x (P(x) \vee Q(x)) \}$,
 $\varphi \equiv \forall x P(x) \vee \forall x Q(x)$.

(b) $\Sigma = \{ \forall x \forall y (R(x, y) \rightarrow R(y, x)), \forall x \forall y \forall z (R(x, y) \wedge R(y, z) \rightarrow R(x, z)) \}$,
 $\varphi \equiv \forall x R(x, x)$.

(c) $\Sigma = \{ \forall x \exists y R(x, y) \}$,
 $\varphi \equiv \exists y \forall x R(x, y)$.