ALETHIC	$\Box\phi$	it is **necessary** that ϕ
	$\Diamond\phi$	it is **possible** that ϕ
DEONTIC	$\Box\phi$	it is **obligatory** that ϕ
	$\Diamond\phi$	it is **permitted** that ϕ
DOXASTIC	$\Box\phi$	it is **believed** that ϕ
TEMPORAL	$F\phi$	it **will be** the case that ϕ
	$P\phi$	it **was** the case that ϕ

1. In EPISTEMIC modal logic $\Box\phi$ reads "it is **known** that ϕ". What is the meaning of $\Diamond\phi$?

2. Formalize the following sentences.

- Necessarily, if snow is white, then snow is white or grass is green.
- I'll go if I must.
- It is possible that Scholz will lose the election.
- Snow might have been either green or blue.
- If snow could have been green, then grass could have been white.
- It is impossible for snow to be both white and not white.
- If grass cannot be clever then snow cannot be furry.
- God's being merciful is inconsistent with your imperfection being incompatible with your going to heaven.

Truth valuational semantics (as in propositional logic) does not work:

1. I am in Kraków.
2. It's possible for me to be in Kraków.
3. 4 is a prime number.
4. It is possible that 4 is a prime number.

Kripke model: (W, R, I) where W = possible worlds, R = accessibility relation, and I = evaluation/interpretation: tells which atomic propositions are true in which worlds. For a world $v \in W$ write $v(\alpha) = 1$ if α is true at v, and $v(\alpha) = 0$ if α is false at v. Rules:

- $v(\sim\alpha) = 1$ iff $v(\alpha) = 0$
- $v(\alpha \wedge \beta) = 1$ iff $v(\alpha) = 1$ and $v(\beta) = 1$
- $v(\alpha \vee \beta) = 1$ iff $v(\alpha) = 1$ or $v(\beta) = 1$
- $v(\alpha \to \beta) = 1$ iff $v(\alpha) = 0$ or $v(\beta) = 1$
- $v(\Diamond\alpha) = 1$ iff there is w such that vRw and $w(\alpha) = 1$
- $v(\Diamond\alpha) = 1$ iff for all w such that vRw, $w(\alpha) = 1$

The formula is **true** (satisfied) **in a model** if it is true at every world.
Modal logic **K**: no restriction on R
Modal logic **S4**: R is transitive and reflexive

1. Check the following formulas on **K** and **S4** models.

1. $\Box p \to p$, $p \to \Diamond p$, $\Box\Box p \to \Box p$, $\Diamond p \to \Diamond\Diamond p$.
2. $\Box p \vee \sim\Box p$, $\Box p \vee \Box \sim p$
3. $\Box p \to \Box \sim\sim p$

2. Show by finding countermodels (models where the formulas are refuted).

1. $\not\models_K \sim(\Box \sim p \to \Box(p \to \sim p))$
2. $\not\models_{S4} \sim((\Diamond p \vee \Box q) \vee \sim\Diamond\Diamond p)$
3. $\not\models_{S4} \sim(\sim\Box(\Box p \to \Box q) \to \sim\Box(p \to q))$