1. Check the following formulas on **K** and **S4** models.

1. $\Box p \to p, \qquad p \to \Diamond p, \qquad \Box\Box p \to \Box p, \qquad \Diamond p \to \Diamond\Diamond p.$
2. $\Box p \vee \sim\Box p, \qquad \Box p \vee \Box \sim p$
3. $\Box p \to \Box \sim\sim p$

1	0
$\alpha \wedge \beta$	
α	
β	

1	0
	$\alpha \wedge \beta$
#1:	α
#2:	β

1	0
$\alpha \vee \beta$	
#1: α	
#2: β	

1	0
	$\alpha \vee \beta$
	α
	β

1	0
$\alpha \to \beta$	
#1:	α
#2: β	

1	0
	$\alpha \to \beta$
α	β

1	0
$\sim\alpha$	
	α

1	0
	$\sim\alpha$
α	

1	0
$\Box\alpha$	
α	

↓ (for all)

1	0
α	

1	0
$\Diamond\alpha$	

↓ (there is)

1	0
α	

1	0
	$\Box\alpha$

↓ (there is)

1	0
	α

1	0
	$\Diamond\alpha$
	α

↓ (for all)

1	0
	α

+reflexive +transitive

Check whether the following formulas are tautologies of $S4$.

1. $\Box p \vee \Box \sim p$
2. $p \to \Diamond p$
3. $\Box(p \vee \sim p)$
4. $\Diamond\Box p \to \Box\Diamond p$
5. $\Box p \to \Box\Box\Box p$
6. $\Box p \to \Box\Box\Diamond p$
7. $\Diamond \sim\sim p \to \Diamond p$

Show that

8. $\Diamond\Diamond p \vdash_{S4} \Diamond p$