Truth tables: $1 = $ TRUE, $0 = $ FALSE.

x	y	$\sim x$	$x \wedge y$	$x \vee y$	$x \to y$	$x \leftrightarrow y$
0	0	1	0	0	1	1
0	1	1	0	1	1	0
1	0	0	0	1	0	0
1	1	0	1	1	1	1

Tautology: φ is a tautology if it is true under **all** evaluations.

Satisfiable: φ is satisfiable if it is true under **some** evaluation.

1. Determine the truth tables of the following formulas. Mark the tautologies.

   - $p \vee \sim p$
   - $p \wedge \sim p$
   - $p \leftrightarrow p$
   - $\sim p \to p$
   - $(p \to q) \vee (q \to p)$
   - $((p \to q) \wedge (q \to r)) \to (p \to r)$
   - $(p \to q) \wedge (r \to q)) \to (p \to r)$
   - $(p \to q) \to (\sim q \to \sim p)$
   - $(p \to q) \to (\sim p \to \sim q)$
   - $(p \to q) \to (q \to p)$
   - $\sim (p \to \sim p)$
   - $(p \to (q \vee r)) \to ((p \to q) \vee (p \to r))$

2. Let $x|y$ be the abbreviation of $\sim (x \wedge y)$. Write up the truth tables of

   - $x|y$
   - $x|x$
   - $(x|x)|(y|y)$
   - $(x|y)|(x|y)$

   Are any of these truth tables resemble to any other operations' truth tables?

3. Two formulas φ and ψ are **equivalent** if they have the same truth tables. Which of the following formulas are equivalent?

   - x
   - $x \wedge y$
   - $\sim x \vee \sim y$
   - $\sim (\sim x \vee \sim y)$
   - $\sim \sim x$
   - $x \to x$
   - $x \to (x \to \sim x)$