1	0
$\alpha \wedge \beta$	
α	
β	

1	0
	$\alpha \wedge \beta$
#1:	α
#2:	β

1	0
$\alpha \vee \beta$	
#1: α	
#2: β	

1	0
	$\alpha \vee \beta$
	α
	β

1	0
$\alpha \to \beta$	
#1:	α
#2: β	

1	0
	$\alpha \to \beta$
α	β

1	0
$\sim\alpha$	
	α

1	0
	$\sim\alpha$
α	

Method: $\varphi \models \psi$ iff applying the algorithm to $\begin{array}{r|l} 1 & 0 \\ \hline \varphi & \psi \end{array}$ leads to a contradiction $\begin{array}{r|l} 1 & 0 \\ \hline p & p \end{array}$ for some variable p.

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1. Which of the following formulas are tautologies? Use the method of analytic tables.
   - (a) $p \vee \sim p$
   - (b) $\sim(p \wedge \sim p)$
   - (c) $(p \to q) \to (\sim q \to \sim p)$
   - (d) $(\sim p \to \sim q) \to (q \to p)$
   - (e) $(p \vee q) \to (p \wedge q)$
2. Formalize and check the following inferences.

   If Charles won the competition, then either Mark came second or Sean came third. Sean didn't come third. Thus, if Mark didn't come second, then Charles didn't win the competition.

   If you play and you study you'll pass the exams, while if you play and don't study you won't pass. Thus, if you play, either you study and you'll pass the exams, or you don't study and you won't pass.
3. More formulas to practice with
   - (a) $(p \to \sim p) \to \sim p$
   - (b) $(\sim p \to p) \to p$
   - (c) $((p \to q) \to p) \to \sim p$
   - (d) $(p \to (p \to q)) \to (p \to \sim q))$
   - (e) $((p \vee q) \wedge \sim q) \to p$
   - (f) $(p \to (q \to p)) \to p$
4. And some more
   - (a) $((p \wedge q) \to r) \to (p \to (q \to r))$
   - (b) $((p \vee \sim q) \to r) \to (p \to (q \to r))$
   - (c) $((p \to q) \to (p \to r)) \to ((p \wedge q) \to r)$
   - (d) $((p \vee q) \wedge (p \to r) \vee (q \to r)) \to r$
   - (e) $((p \wedge \sim q) \to \sim r) \to ((p \vee r) \to q)$
   - (f) $((p \wedge q) \to r) \to ((p \wedge \sim r) \to \sim q)$
   - (g) $((\sim p \to (\sim q \to r)) \wedge \sim r) \to (q \vee p)$
   - (h) $((p \wedge q) \to r) \to ((p \wedge \sim r) \to \sim q)$
   - (i) $(p \to (q \to \sim r)) \to ((p \to q) \to (r \to \sim p))$