Semantics of intuitionistic logic: An intuitionistic frame is a tuple $\mathcal{F} = (W, R)$ where W is a non-empty set, and R is a binary relation which is reflexive, transitive and antisymmetric. **Intuitionistic evaluations**: for each world $w \in W$ and propositional letter p we decide whether or not p is true at w: $w(p) = 1$ or $w(p) = 0$; and we pay attention that the intuitionistic evaluations should be **upward closed**: if $w(p) = 1$ and wRv, then $v(p) = 1$. A frame together with and intuitionistic evaluation is called a model (intuitionistic model). Then we extend the evaluation to formulas inductively by

- $w(\phi \wedge \psi) = 1$ if and only if $w(\phi) = 1$ and $w(\psi) = 1$,
- $w(\phi \vee \psi) = 1$ if and only if $w(\phi) = 1$ or $w(\psi) = 1$,
- $w(\phi \to \psi) = 1$ if and only if for all v such that wRv, we have (if $v(\phi) = 1$, then $v(\psi) = 1$),
- $w(\sim\phi) = 1$ if and only if for all v such that wRv, we have $v(\phi) = 0$.

Note that $w(\phi) = 0$ and $w(\sim\phi) = 1$ are *not* equivalent (unlike in propositional logic) because it might happen that $w(\phi) = 0$ but still there is v such that wRv and $v(\phi) = 1$. A formula is an **intuitionistic tautology** if it is true in every world of every intuitionistic model.

1. Let us discuss what reflexivity, transitivity, and antisymmetry are. Examples, counterexamples.

2. Examples, counterexamples for intuitionistic evaluations.

3. Let us calculate the truth values of some random formulas in random intuitionistic models.

4. Let us see an example for that $w(\phi) = 0$ and $w(\sim\phi) = 1$ are not the same.

5. Take
$$W = \{v, w\}, \qquad R = \{(v, v), (v, w), (w, w)\}, \qquad v(p) = 0, \quad w(p) = 1\,.$$
Draw the model. Calculate $v(p \vee \sim p)$ and $w(p \vee \sim p)$.

6. Check that $p \to p$ is an intuitionistic tautology. Check it on one model first, and try to generalize.

7. Check whether $p \to \sim\sim p$ is an intuitionistic evaluation.