First-order logic (Klasyczny Rachunek Predykatów, KRP).

(1) Which of the following strings are well formed formulas of first-order logic? In each case determine the role of the symbols.
 (a) $\exists x \forall y (R(x, y, z))$, $\exists x \forall x (R(x, x))$, $\exists \forall y (x = y)$, $(\exists x \wedge y)(x \neq y)$
 (b) $\exists \neg x (x = x)$, $\forall x R(x, \exists y)$, $\forall y \exists x (x = f(y) \wedge R(f(f(y)), x))$
 (c) $\forall y \exists x (R(x, y) \wedge y = R(x))$

(2) Formalize the sentences below in a suitable first-order language.
 (a) A small, happy dog is at home. Every small dog that is at home is happy.
 (b) Anne introduced Brian to Cecile. Anne introduced Brian to everyone.
 (c) Romeo loves Giulietta only.
 (d) If you praise everybody, you praise nobody.
 (e) Two people were waiting for the tram.
 (f) Everyone's paternal grandfather is nicer than their father.
 (g) There is a barber who shaves all those, and those only, who do not shave themselves.[1]

(3) Express the following quantifiers in first-order logic (n is a natural number).

- $\exists_{\geq n} x\, \phi(x)$ = "there are at least n x's such that $\phi(x)$"
- $\exists_{\leq n} x\, \phi(x)$ = "there are at most n x's such that $\phi(x)$"
- $\exists_{= n} x\, \phi(x)$ = "there are exactly n x's such that $\phi(x)$"

Can we express

- $\exists_{\infty} x\, \phi(x)$ = "there are infinitely many x's such that $\phi(x)$" ?

[1] Does this barber shave himself?