Model: $\mathcal{A} = \langle A, \{R^{\mathcal{A}} : R \in Rel\}, \{f^{\mathcal{A}} : f \in Fun\}, \{c^{\mathcal{A}} : c \in Const\}\rangle$, where

- $A \neq \emptyset$ is an arbitrary set,
- For a constant symbol c we have $c^{\mathcal{A}} \in A$,
- For a function symbol f of arity n we have $f^{\mathcal{A}} : A^n \to A$ is an n-place function,
- For a relation symbol R of arity n we have $R^{\mathcal{A}} \subseteq A^n$ is an n-place relation.

Our convention is to denote models by calligraphic letters and their universes with the same but capital letters

Evaluation: If $\mathcal{A}$ is a structure, then a function $e : Var \to A$ is called an evaluation (over A). We define the value $\tau^{\mathcal{A}}[e]$ of a term τ by recursion as follows.

- $v^{\mathcal{A}}[e] = e(v)$ for $v \in Var$,
- $c^{\mathcal{A}}[e] = c^{\mathcal{A}}$ for a constant symbol c,
- $(f(\tau_1, \ldots, \tau_n))^{\mathcal{A}}[e] = f^{\mathcal{A}}(\tau_1^{\mathcal{A}}[e], \ldots, \tau_n^{\mathcal{A}}[e])$ for a function symbol f of arity n and terms τ_i.

Satisfaction: For a structure $\mathcal{A}$, evaluation $e : Var \to A$ and formula φ we define $\mathcal{A} \models \varphi[e]$ by recursion as follows.

- $\mathcal{A} \models R(\tau_1, \ldots, \tau_n)[e]$ if and only if $\langle \tau_1^{\mathcal{A}}[e], \ldots, \tau_n^{\mathcal{A}}[e]\rangle \in R^{\mathcal{A}}$,
- $\mathcal{A} \models (\tau = \sigma)[e]$ if and only if $\tau^{\mathcal{A}}[e] = \sigma^{\mathcal{A}}[e]$,
- $\mathcal{A} \models \neg\varphi[e]$ if and only if it is not the case that $\mathcal{A} \models \varphi[e]$,
- $\mathcal{A} \models (\varphi \wedge \psi)[e]$ if and only if $\mathcal{A} \models \varphi[e]$ and $\mathcal{A} \models \psi[e]$,
- $\mathcal{A} \models \exists v\varphi[e]$ if and only if there is $a \in A$ such that $\mathcal{A} \models \varphi[e, v = a]$, where $[e, v = a]$ is the evaluation which can differ from e only at $v \in V$ and assigns the value $a \in A$ to v.

Truth: $\mathcal{A} \models \phi$ if for all evaluations $e : Var \to A$ we have $\mathcal{A} \models \phi[e]$. If Γ is a set of formulas, then $\mathcal{A} \models \Gamma$ if $\mathcal{A} \models \varphi$ for all $\varphi \in \Gamma$.

---

1. Let's check examples for structures, evaluations of terms, satisfaction of formulas, etc.

2. Give a structure $\mathcal{M}$ and a formula φ such that neither $\mathcal{M} \models \varphi$ nor $\mathcal{M} \models \neg\varphi$ holds.

3. Which of the following statements are true?
   (a) $\mathbb{Q} \models \forall x \forall y \exists z (x < z < y)$,
   (b) $\mathbb{Q} \models \exists y (y^2 + 1 = 0)$,
   (c) $\mathbb{Q} \models [\forall y (y > 0)] \to [\exists y (y^2 + 1 = 0)]$,
   (d) $\mathbb{R} \models [\forall y (y > 0)] \to [\exists y (y^2 + 1 = 0)]$,

4.* Consider the structure $\mathcal{M} = \langle \mathbb{N}, 0, 1, +, \cdot, \leq \rangle$ with the usual interpretation of the symbols. Find formulas $\varphi(x)$ and $\psi(x)$ which hold for (a) prime numbers and (b) powers of two only.