

1. Define an appropriate language and formalize the following sentences in first-order logic:

- A is above C, D is on E and above F.
- A is green, while C is not.
- Everything is on something.
- Everything that is free has nothing on it.
- Everything that is green is free.
- There is something that is red and is not free.
- Everything that is not green and is above B, is red.

2. Let us set the language as follows: Constant symbols = $\{A, B, C, D, E, F\}$,

Predicates = $\{On(x, y), Above(x), Free(x), Red(x), Green(x)\}$.

Let us define the model $\mathcal{M}$ with the interpretations:

$$A^{\mathcal{M}} = b_1, \quad B^{\mathcal{M}} = b_2, \quad C^{\mathcal{M}} = b_3, \quad D^{\mathcal{M}} = b_4, \quad E^{\mathcal{M}} = b_5, \quad F^{\mathcal{M}} = table$$

$$On^{\mathcal{M}} = \{(b_1, b_4), (b_4, b_3), (b_3, table), (b_5, b_2), (b_2, table)\}$$

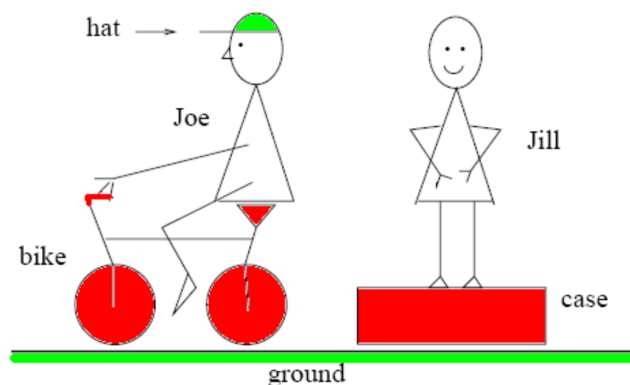
$$Above^{\mathcal{M}} = \{(b_1, b_4), (b_1, b_3), (b_1, table), (b_4, b_3), (b_4, table), (b_3, table), (b_5, b_2), (b_5, table), (b_2, table)\}$$

$$Free^{\mathcal{M}} = \{b_1, b_5\}, \quad Green^{\mathcal{M}} = \{b_4\}, \quad Red^{\mathcal{M}} = \{b_1, b_5\}$$

(a) Draw the model.

The picture below illustrates the model $\mathcal{N}$, where

$$A^{\mathcal{N}} = hat, \quad B^{\mathcal{N}} = Joe, \quad C^{\mathcal{N}} = bike, \quad D^{\mathcal{N}} = Jill, \quad E^{\mathcal{N}} = case, \quad F^{\mathcal{N}} = ground$$



(b) Write up formally the interpretations of the predicates in $\mathcal{N}$. (c) For each formula from Exercise 1, decide whether it is satisfied in $\mathcal{M}$ or $\mathcal{N}$.

3. Which of the following statements are true? $\mathbb{Q}$ is the set of rational numbers, and $<^{\mathbb{Q}}$ is the usual ordering among the rational numbers.

- (a) $\mathbb{Q} \models \forall x \forall y \exists z (x < z < y)$,
- (b) $\mathbb{Q} \models \exists y (y^2 + 1 = 0)$,
- (c) $\mathbb{Q} \models [\forall y (y > 0)] \rightarrow [\exists y (y^2 + 1 = 0)]$,
- (d) $\mathbb{R} \models [\forall y (y > 0)] \rightarrow [\exists y (y^2 + 1 = 0)]$,

4. Find models and countermodels for the following sets of formulas.

- (a) $\forall x \exists y \exists z (E(x, y) \wedge \neg E(x, z))$;
- (b) $\forall x \forall y (x \neq y \rightarrow \exists z (E(x, z) \wedge \neg E(y, z)))$;
- (c) $\forall x (\exists y E(x, y) \rightarrow P(x))$;
- (d) $\exists x \forall y (f(x, y) = g(y))$.