1. Let $A = \{1, 3, 5, 15\}$ and $E^{\mathcal{A}} =$"being even", $M^{\mathcal{A}} =$"being a multiple of", $L^{\mathcal{A}} =$"being less than". Let $a^{\mathcal{A}} = 1$, $b^{\mathcal{A}} = 3$, $c^{\mathcal{A}} = 5$ and $d^{\mathcal{A}} = 15$. Determine whether the following formulas are satisfied in $\mathcal{A}$.
   (a) $\exists y E(y)$
   (b) $\forall x \sim E(x)$
   (c) $\forall x M(x, a)$
   (d) $\forall x M(x, b)$
   (e) $\exists x M(x, d)$
   (f) $\exists x L(x, a)$
   (g) $\forall x(E(x) \to M(x, a))$
   (h) $\forall x \exists y L(x, y)$
   (i) $\forall x \exists y M(x, y)$
   (j) $\forall x(M(x, b) \to L(x, c))$
   (k) $\forall x \forall y(L(x, y) \to \sim L(y, x))$
   (l) $\forall x(M(x, c) \vee L(x, c))$

2. Find models and counter-models for the following sets of formulas.
   (a) $\forall x(\exists y E(x, y) \to P(x))$;
   (b) $\exists x \forall y(f(x, y) = g(y))$.
   (c) $\big(\forall x \exists y(x < y)\big) \ \wedge \ \big(\forall x \exists y(y < x)\big)$
   (d) $\forall x \forall y \exists z(x < z \ \wedge \ z < y)$

3. Show that φ is not a consequence of Γ, where
   (a) $\Sigma = \{ \ \forall x(P(x) \vee Q(x)) \ \}$,
   $\varphi \equiv \forall x P(x) \vee \forall x Q(x)$.
   (b) $\Sigma = \{ \ \forall x \forall y(R(x, y) \to R(y, x)), \forall x \forall y \forall z(R(x, y) \wedge R(y, z) \to R(x, z)) \ \}$,
   $\varphi \equiv \forall x R(x, x)$.
   (c) $\Sigma = \{ \ \forall x \exists y R(x, y) \ \}$,
   $\varphi \equiv \exists y \forall x R(x, y)$.