Derivations. Briefly: use anything from propositional logic + new rules (that we discuss one by one). The first such new rule is:

> $\forall Out$: If x is **free** in $\varphi(x)$, then
>
> $$\frac{\forall x \varphi(x)}{\varphi[x/t]}$$

(1) Let c be a constant. Check if the $\forall Out$ rule can be applied. If so, what is the result?

- (a) $\forall x F(x)$
- (b) $\forall x(F(x) \rightarrow G(x))$
- (c) $\forall x(F(x) \rightarrow \forall x G(x))$
- (d) $\sim\forall x G(x)$
- (e) $\forall x F(x) \rightarrow \forall x G(x)$
- (f) $\forall x \exists y R(x, y)$
- (g) $\forall x R(x, x)$

(2) Construct derivations using the $\forall Out$ rule.

$$\frac{\forall x(F(x) \rightarrow H(x)) \quad F(c)}{H(c)}$$

$$\frac{\forall x(S(x) \rightarrow P(x)) \quad \forall x((S(x) \wedge P(x)) \rightarrow D(x)) \quad S(m)}{D(m)}$$

(3) Derive

$$\frac{\forall x F(x) \rightarrow \forall x G(x)}{\forall x F(x) \rightarrow G(a)}$$

$$\frac{\forall x(F(x) \rightarrow H(x)) \quad \sim H(b)}{\sim\forall x F(x)}$$

$$\frac{\forall x(F(x) \rightarrow \forall y R(x, y)) \quad \forall x \forall y(R(x, y) \rightarrow \forall z G(z)) \quad \sim G(b)}{\sim F(a)}$$

> $\exists In$: If $\varphi[x/t]$ is a substitution instance of $\varphi(x)$, then
>
> $$\frac{\varphi[x/t]}{\exists x \varphi(x)}$$

(4) Apply the $\exists In$ rule in all possible ways to the formulas below (if possible).

- (a) $F(b)$
- (b) $R(c, d)$
- (c) $F(c) \wedge G(c)$

(5) Derive

$$\frac{\forall x(F(x) \rightarrow H(x)) \quad F(a)}{\exists x H(x)}$$

$$\frac{\forall x(G(x) \rightarrow H(x)) \quad G(b)}{\exists x(G(x) \wedge H(x))}$$

$$\frac{\exists x \sim R(x, a) \rightarrow \sim\exists x R(a, x) \quad \sim R(a, a)}{\sim R(a, b)}$$