Derivations. Briefly: use anything from propositional logic + new rules (that we discuss one by one). The first such new rule is:

$\forall Out$: If x is **free** in $\varphi(x)$, then

$$\frac{\forall x \varphi(x)}{\varphi[x/t]}$$

$\exists In$: If $\varphi[x/t]$ is a substitution instance of $\varphi(x)$, then

$$\frac{\varphi[x/t]}{\exists x \varphi(x)}$$

(1) Apply the $\exists In$ rule in all possible ways to the formulas below (if possible).

   (a) $F(b)$

   (b) $R(c, d)$

   (c) $F(c) \wedge G(c)$

(2) Derive

$$\frac{\forall x(F(x) \to H(x)) \quad F(a)}{\exists x H(x)} \qquad \frac{\forall x(G(x) \to H(x)) \quad G(b)}{\exists x(G(x) \wedge H(x))} \qquad \frac{\exists x \sim R(x,a) \to \sim \exists x R(a,x) \quad \sim R(a,a)}{\sim R(a,b)}$$

K:

$$\frac{\forall x(\varphi(x) \to \psi(x))}{\forall x \varphi(x) \to \forall x \psi(x)}$$

Gen: If x is *not* free in φ, then

$$\frac{\varphi}{\forall x \varphi(x)}$$

(3) Derive

$$\frac{\forall x(F(x) \to G(x)) \quad \forall x F(x)}{\forall x G(x)} \qquad \frac{\forall x(F(x) \to G(x)) \quad \forall x(G(x) \to H(x))}{\forall x(F(x) \to H(x))}$$

(4) Which of the following formulas are logical truth?

   (a) $\exists x \forall y \varphi \to \forall y \exists x \varphi$

   (b) $\forall x \exists y \varphi \to \exists y \forall x \varphi$

   (c) $\forall x \exists x \varphi \to \exists x \forall x \varphi$