1. What is $\mathcal{P}(\{\emptyset, \{\emptyset\}\})$?

2. Determine $\mathcal{P}(X \cup Y) \cap \mathcal{P}(Y)$ where $X = \{a, b\}$ and $Y = \{b, c\}$

3. The definition of an ordered pair, given by Kuratowski, is

$$(a, b) = \{\{a\}, \{a, b\}\}.$$

   Show that $(a, b) = (c, d)$ if and only if $a = c$ and $b = d$.

4. Does the following alternative definitions of "ordered pairs" have the same property?

$$\begin{aligned}
(a,b)_1 &= \{\{a\}, \{\{b\}\}\} \\
(a,b)_2 &= \{\{a\}, b\} \\
(a,b)_3 &= \{a, \{a, b\}\} \\
(a,b)_W &= \{\{\{a\}, \emptyset\}, \{\{b\}\}\} && \text{(Norbert Wiener)} \\
(a,b)_H &= \{\{a, 0\}, \{b, 1\}\} && \text{(Felix Hausdorff)}
\end{aligned}$$

5. An equivalence relation is a binary relation R that is reflexive, transitive and symmetric. Show that the relation $(a, b) \sim (c, d)$ iff $a - c = b - d$ is an equivalence relation over the pairs of integers.

6. For an equivalence relations $\sim$ over the set X, and an element $x \in X$ let us write

$$x/_\sim = \{y \in X : \; x \sim y\}, \qquad X/_\sim = \{x/_\sim : \; x \in X\}.$$

   ($x/_\sim$ is called the equivalence class of x). Show that

   (a) If $x \sim y$, then $x/_\sim = y/_\sim$.

   (b) If $x \not\sim y$, then $x/_\sim$ and $y/_\sim$ are disjoint.

   (c) $\bigcup_{x \in X} x/_\sim = X$.

7. For $x, y \in \mathbb{Z}$ let $x \sim y$ if and only if $x - y$ is even. What does $\mathbb{Z}/_\sim$ look like? How many elements does it have?

8. Let $f : X \to Y$ be surjective. Show that $\ker(f) = \{(a, b) : \; f(a) = f(b)\}$ is an equivalence relation on X. Can you find X, Y and f such that $\ker(f)$ is the relation $\sim$ from the previous exercise?