(1) Give the definitions of the following concepts: model for a first-order language; valuation; satisfaction of formulas (atomic and compound) in a model and with respect to a valuation; truth of a formula in a model; when a formula is a semantic consequence of a set of formulas.

Formalize the following sentences in first-order logic and check whether the premises imply the conclusion. If not, give a refuting model.

Premises

Every dog is a mammal.

No mammal is a fish.

Some animals are fish.

Conclusion

Some animals are not dogs.

(2) Give the definitions of: valuation in propositional logic (KRZ); frame and valuation in intuitionistic logic (INT); logical truth in KRZ and INT.

Please check whether the sentence

> "If it is not true that (I ate lunch and drank coffee), then (I did not eat lunch or drink coffee)"

is a logical truth of KRZ and INT. If the answer is no, please provide a refuting interpretation.

(3) The following formulas are given:

a. $(\neg p \to \neg q) \to (p \to q)$

b. $(p \to (\neg q \to \neg r)) \to ((p \wedge \neg q) \to \neg r)$

c. $\neg((p \to \neg p) \wedge (\neg p \to p))$

For each formula that is a theorem of propositional logic, give a derivation. For each formula that is not a theorem of propositional logic, please justify why it is not so.

(4) Give the definitions of: model of $S4$; tautology in $S4$. Check whether the following formulas are tautologies and if not, give an $S4$-model that refutes the formula.

a. $\Diamond p \vee \Diamond \neg p$

b. $\Diamond(p \vee \neg p)$