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Symbols

$[X]^\kappa, [X]^{<\kappa}$ —family of subsets, 1
 \rightleftarrows —back and forth, 84
 $\lceil \varphi \rceil$, 112
 μ —operator, 19, 27
 Ω —set of functions, 19
 Ω^* —partial functions, 27
 $\wp(X)$ —powerset, 1
 Σ^* —set of words over Σ , 13
 $\Sigma_n, \Pi_n, \Delta_n$ —arithmetical hierarchy, 120
 $\alpha(o)$ —code of an object, 29, 50, 63
 β —Gödel's β function, 23
 \equiv —elementary equivalence, 77
 \exists_n —formula, 78
 \forall_n —formula, 78
 κ -almost disjoint, 2
 κ -categorical, 80
 κ -compact, 100
 κ -saturated, 100
 \models —semantical consequence, 42, 55
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 ω -model, 76
 ω -rule, 76
 ω_1^{CK} —Church–Kleene ordinal, 37
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 $<$ —elementary substructure, 78
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